Efficient subset and superset queries

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Abstract. The paper presents a data structure for storing and querying sets called SetTrie. Besides the operations insert and search defined for ordinary tries, we introduce the operations for retrieving subsets and supersets of a given set from a SetTrie tree. The performance of operations is analysed empirically in a series of experiments. The analysis shows that sets can be accessed in $O(c \times |set|)$ time where $c$ is up to 5 for subset case and approximately 150 in average case for the superset case.

Keywords. Containment queries, Indexes, Access methods, Databases

1. Introduction

Let $U$ be a set of ordered symbols. The subsets of $U$ are denoted as words. Given a set of words $S$ and a subset of $U$ named $X$, we are interested in the following queries.

1. Is $X$ a subset of any element from $S$?
2. Is $X$ a superset of any element from $S$?
3. Enumerate all $Y$ in $S$ such that $X$ is a subset of $Y$.
4. Enumerate all $Y$ in $S$ such that $X$ is a superset of $Y$.

Let us show a simple example. Let $U = \{1, 2, 3, ..., 10\}$ and $S = \{\{2, 3, 5, 7\}, \{2, 5, 9, 10\}, \{5, 7\}, \{3, 5, 7, 8\}, \{5, 6, 7, 9\}\}$. Given the set $X = \{3, 5, 7\}$, the supersets of $X$ in $S$ are $\{2, 3, 5, 7\}$ and $\{3, 5, 7, 8\}$. The set $\{5, 7\}$ is the single subset of $X$ in $S$.

The paper presents a data structure SetTrie that implements efficiently the above stated queries. SetTrie is a tree data structure similar to trie [4]. The possibility to extend the performance of usual trie from membership operation to subset and superset operations comes from the fact that we are storing sets and not the sequences of symbols as for ordinary tries.

Since we are dealing with sets for which the ordering of the elements is not important, we can define a syntactical order of symbols by assigning each symbol a unique index. Words are ordered by sequences of indices. The ordering of words is exploited for the representation of sets of words as well as in the implementation of the above stated operations.

SetTrie is a tree storing a set of words which are represented by a path from the root of SetTrie to a node corresponding to the indices of elements from words. As with tries, prefixes that overlap are represented by a common path from the root to an internal vertex of SetTrie tree.
Figure 1. Example of SetTrie

The operations for searching subsets and supersets of a set $X$ in $S$ use the ordering of $U$. The algorithms do not need to consider the tree branches for which we know they do not lead to results on the basis of the ordering of word symbols. The search space for a given $X$ and tree representing $S$ can be seen as a subtree determined primarily by the search word $X$ but also with the search tree corresponding to $S$.

We test the operations in two types of experiments. Firstly, we examine the execution of the operations on real-world data where sets represents words from the English dictionary. Secondly, we have tested the operations on artificially generated data. In these experiments we tried to see how three main parameters: the size of words, the size of SetTrie tree and the size of test-set, affect the behaviour of the operations.

The paper is organized as follows. The following section presents the data structure SetTrie together with the operations for searching the subsets and supersets in a tree. The Section 3 describes the empirical study of SetTrie. We present a series of experiments that measure the behaviour of operations and the size of data structure. The related work is presented in Section 4. Finally, the overview and some conclusions are given in Section 5.

2. Data structure SetTrie

SetTrie is a tree composed of nodes labelled with indices from 1 to $N$ where $N$ is the size of the alphabet. The root node is labelled with $\{}$ and its children can be the nodes labelled from 1 to $N$. A root node alone represents an empty set. A node labelled $i$ can have children labelled with numbers greater than $i$. Each node can have a flag denoting the last element in the set. Therefore, a set is represented by a path from the root node to a node with flag set to true.

Let us give an example of SetTrie. Figure 2 presents a SetTrie containing the sets $\{1, 3\}$, $\{1, 3, 5\}$, $\{1, 4\}$, $\{1, 2, 4\}$, $\{2, 4\}$, $\{2, 3, 5\}$. Note that flagged nodes are represented with circles.

2.1. Operations

Let us first present a data structure for storing words, that is, the sets of symbols. Words are stored in a data structure Word representing ordered sets of integer numbers.

The users of Word can scan sets using the following mechanism. The operation `word.gotoFirstElement` sets the current element of word to the first element of ordered
set. Then, the operation \( \text{word.existsCurrentElement} \) checks if \( \text{word} \) has the current element set. The operation \( \text{word.currentElement} \) returns the current element, and the operation \( \text{word.gotoNextElement} \) goes to the next element in the set.

Let us now describe the operations of the data structure \( \text{SetTrie} \). The first operation is insertion. The operation \( \text{insert(root,word)} \) enters a new \( \text{word} \) into the \( \text{SetTrie} \) referenced by the root \( \text{node} \). The operation is presented by Algorithm 1.

**Algorithm 1** \( \text{insert(node, word)} \)

1. if \((\text{word.existsCurrentElement})\) then
2.  \(\text{if (exists child of node labelled word.currentElement)}\) then
3.   \(\text{nextNode = child of node labelled word.currentElement};\)
4.  \(\text{else}\)
5.   \(\text{nextNode = create child of node labelled word.currentElement};\)
6.  \(\text{end if}\)
7.  \(\text{insert(nextNode, word.gotoNextElement)}\)
8. else
9.  \(\text{node’s flag_last = true};\)
10. \(\text{end if}\)

Each invocation of operation \( \text{insert} \) either traverses through the existing tree nodes or creates new nodes to construct a path from the root to the flagged node corresponding to the last element of the ordered set.

The following operation \( \text{search(node, word)} \) searches for a given \( \text{word} \) in the tree \( \text{node} \). It returns true when it finds all symbols from the word, and false as soon one symbol is not found. The algorithm is shown in Algorithm 2. It traverses the tree \( \text{node} \) by using the elements of ordered set \( \text{word} \) to select the children.

**Algorithm 2** \( \text{search(node, word)} \)

1. if \((\text{word.existsCurrentElement})\) then
2.  \(\text{if (there exists child of node labelled word.currentElement)}\) then
3.   \(\text{matchNode = child vertex of node labelled word.currentElement};\)
4.  \(\text{search(matchNode, word.gotoNextElement)};\)
5.  \(\text{else}\)
6.   \(\text{return false};\)
7.  \(\text{end if}\)
8. else
9.  \(\text{return (node’s last_flag == true) } ;\)
10. \(\text{end if}\)

Let us give a few comments to present the algorithm in more detail. The operation have to be invoked with the call \( \text{search(root,set.gotoFirstElement)} \) so that \( \text{root} \) is the root of the \( \text{SetTrie} \) tree and the current element of the \( \text{word} \) is the first element of \( \text{word} \). Each activation of \( \text{search} \) tries to match the current element of \( \text{word} \) with the child of \( \text{node} \). If the match is not successful it returns \( \text{false} \) otherwise it proceeds with the following element of \( \text{word} \).

The operation \( \text{existsSubset(node,word)} \) checks if there exists a subset of \( \text{word} \) in the given tree referenced by \( \text{node} \). The subset that we search in the tree has fewer
elements than word. Therefore, besides that we search for the exact match we can also skip one or more elements in word and find a subset that matches the rest of the elements of word. The operation is presented in Algorithm 3.

Algorithm 3 existsSubset(node, set)
1: if (node.last_flag == true) then
2:    return true;
3: end if
4: if (not word.existsCurrentElement) then
5:    return false;
6: end if
7: found = false;
8: if (node has child labelled word.currentElement) then
9:    nextNode = child of node labelled word.currentElement;
10:   found = existsSubset(nextNode, word.gotoNextElement);
11: end if
12: if (!found) then
13:    return existsSubset(node, word.gotoNextElement);
14: else
15:    return true;
16: end if

Algorithm 3 tries to match elements of word by descending simultaneously in tree and in word. The first IF statement (line 1) checks if a subset of word is found in the tree i.e. the current node of a tree is the last element of subset. The second IF statement (line 4) checks if word has run of the elements. The third IF statement (line 8) verifies if the parallel descend in word and tree is possible. In the positive case, the algorithm calls existsSubset with the next element of word and a child of node corresponding to matched symbol. Finally, if match did not succeed, current element of word is skipped and existsSubset is activated again in line 13.

The operation existsSubset can be easily extended to find all subsets of a given word in a tree node. After finding the subset in line 15 the subset is stored and the search continues in the same manner as before. The experimental results with the operation getAllSubsets(node, word) are presented in the following section.

The operation existsSuperset(node, word) checks if there exists a superset of word in the tree referenced by node. While in operation existsSubset we could skip some elements from word, here we can do the opposite: the algorithm can skip some elements in supersets represented by node. Therefore, word can be matched with the subset of superset from a tree. The operation is presented in Algorithm 4.

Let us present Algorithm 4 in more detail. The first IF statement checks if we are already at the end of word. If so, then the parameter word is covered completely with a superset from tree. Lines 5-6 set the lower and upper bounds of iteration. In each pass we either skip current child and call existsSuperset on unchanged word (line 11), or, descend in parallel on both word and tree in the case that we reach the upper bound i.e. the next element in word (line 9).

Again, the operation existsSuperset can be quite easily extended to retrieve all supersets of a given word in a tree node. However, after word (parameter) is matched
Algorithm 4 existsSuperset(node, word)
1: if (not word.existsCurrentElement) then
2:    return true;
3: end if
4: found = false;
5: from = word.currentElement;
6: upto = word.nextElement if it exists and N otherwise;
7: for (each child of node labelled l: from < l ≤ upto) while !found do
8:    if (child is labelled upto) then
9:        found = existsSuperset(child, word.gotoNextElement);
10:   else
11:       found = existsSuperset(child, word);
12:   end if
13: end for

completely (line 2 in Algorithm 4), there remains a subtree of trailers corresponding to a set of supersets that subsume word. This subtree is rooted in a tree node, let say node$_k$, that corresponds to the last element of word. Therefore, after the node$_k$ is matched against the last element of the set in line 2, the complete subtree has to be traversed to find all supersets that go through node.

3. Experiments

The performance of the presented operations is analysed in four experiments. The main parameters of experiments are: the number of words in the tree, the size of the alphabet, and the maximum length of words. The parameters are named: numTreeWord, alphabetSize, and maxSizeWord, respectively. In every experiment we measure the number of visited nodes necessary for an operation to terminate.

In the first experiment, SetTrie is used to store real-world data – it stores the words from English Dictionary. In the following three experiments we use artificial data – datasets and test data are randomly generated. In these experiments we analyse in detail the interrelations between one of the stated tree parameters on the number of visited nodes.

In all experiments we observe four operations presented in the previous section: existsSubset (abbr. esb) and its extension getAllSubsets (abbr. gsb), and existsSuperset (abbr. esr) and its extension getAllSupersets (abbr. gsr).

3.1. Experiment with real-world data

Let us now present the first experiment in more detail. The number of words in test set is 224,712 which results in a tree with 570,462 nodes. The length of words are between 5 and 24 and the size of the alphabet (alphabetSize) is 25. The test set contains 10,000 words.

Results are presented in Table 1 and Figure 2. Since there are 10,000 words and 23 different word lengths in the test set, approximately 435 input words are of the same length. Table 1 and Figure 2 present the average number of visited nodes for each input word length (except for gsr where values below word length 6 are intentionally cut off).
Let us give some comments on the results presented in Table 2. First of all, we can see that the superset operations (esr and gsr) visit more nodes than subset operations (esb and gsb).

The number of nodes visited by esr and gsr decreases as the length of words increases. This can be explained by more constrained search in the case of longer words, while it is very easy to find supersets of shorter words and, furthermore, there are a lot of supersets of shorter words in the tree.

Since operation gsr returns all supersets (of a given set), it always visits more nodes than the operation esr. However, searching for the supersets of longer words almost always results in failure and for this reason the number of visited nodes is the same for both operations.

The number of visited nodes for esb in the case that words have more than 5 symbols is very similar to the length of words. Below this length of words both esb and gsb visit the same number of nodes, because there were no subset words of this length in the tree and both operations visit the same nodes.

The number of visited nodes for gsb linearly increases as the word length increases. We have to visit all the nodes that are actually used for the representation of all subsets of a given parameter set.

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Figure 2. Visited nodes for dictionary words
3.2. Experiments with artificial data

In experiment 1 we observe the influence of changing the maximal length of word to the performance of all four operations. We created four trees with alphabetSize 30 and numTreeWord 50,000. maxSizeWord is different in each tree: 20, 40, 60 and 80, for tree1, tree2, tree3 and tree4, respectively. The length of word in each tree is evenly distributed between the minimal and maximal word size. The number of nodes in the trees are: 332,182, 753,074, 1,180,922 and 1,604,698. The test set contains 10,000 words.

Figure 3 shows the performance of all four operations on all four trees. The performance of superset operations is affected more by the change of the word length than the subset operations.

With an even distribution of data in all four trees, esr visits most nodes for input word lengths that are about half of the size of maxSizeWord (as opposed to dictionary data where it visits most nodes for word lengths approximately one fifth of maxSizeWord). For word lengths equal to maxSizeWord the number of visited nodes is roughly the same for all trees, but that number increases slightly as the word length increases.

esb operation visits fewer than 10 nodes most of the time, but for tree3 it goes up to 44 which is still a very low number. The experiment was repeated multiple (about 10) times, and in every run the operation "jumped up" in a different tree. As seen later in experiment 2, it seems that numTreeWord 50 is just on the edge of the value where esb stays constantly below 10 visited nodes. It is safe to say that the change in maxSizeWord has no major effect on existsSubSet operation.

In contrast to gsr, gsb visits less nodes for the same input word length in trees with greater maxSizeWord, but the change is minimal. For example for word length 35 in tree2 (maxSizeWord 40) gsb visits 7,606 nodes, in tree3 (maxSizeWord 60) it visits 5,300 nodes and in tree4 (maxSizeWord 80) it visits 4,126 nodes.

In experiment 2 we are interested about how a change in the number of words in the tree affects the operations. Ten trees are created all with alphabetSize 30 and maxSizeWord 30. numTreeWord is increased in each tree by 10,000 words: tree1 has 10,000 words, and tree10 has 100,000 words. The number of nodes in the trees (from
Experiment 1 - increasing `maxSizeWord`

`tree1` to `tree10` are: 115,780, 225,820, 331,626, 437,966, 541,601, 644,585, 746,801, 846,388, 946,493 and 1,047,192. The test set contains 5,000 words.

Figure 4 shows the number of visited nodes for each operation on four trees: `tree1`, `tree4`, `tree7` and `tree10` (only every third tree is shown to reduce clutter). When increasing `numTreeWord` the number of visited nodes increases for `esr`, `gsr` and `gsb` operations. `esb` is least affected by the increased number of words in the tree. In contrast to the other three operations, the number of visited nodes decreases when `numTreeWord` increases.

For input word lengths around half the value of `maxSizeWord` (between 13 and 17) the number of visited nodes for `esr` increases with the increase of the number of words in the tree. For input word lengths up to 10, the difference between trees is minimal. After word lengths about 20 the difference in the number of visited nodes between trees starts to decline. Also, trees 7 to 10 have very similar results. It seems that after a certain number of words in the tree the operation "calms down".

The increased number of words in the tree affects the `gsr` operation mostly in the first quarter of `maxSizeWord`. The longer the input word, the lesser the difference between trees. Still, this operation is the most affected by the change of `numTreeWord`. The average number of visited nodes for all input word lengths in `tree1` is 8,907 and in `tree10` it is 68,661. Due to the nature of the operation, this behaviour is expected. The more words there are in the tree, the more supersets can be found for an input word.
As already noted above, when the number of words in the tree increases the number of visited nodes for esb decreases. After a certain number of words, in our case this was around 50,000, the operation terminates at a minimum possible visits of nodes for any word length. The increase of numTreeWord seems to "push down" the operation from left to right. This can be seen in figure 4 by comparing tree1 and tree4. In tree1 the operation visits more than 10 after word length 15, and in tree4 it visits more than 10 nodes after word length 23. Overall the number of visited nodes is always very low.

The chart of gsb operation looks like a mirrored chart of gsr. The increased number of words in the tree has more effect on input word lengths where the operation visits more nodes (longer words). Below word length 15 the difference between trees is in the range of 100 visited nodes. At word length 30 gsb visits 1,729 nodes in tree1 and 8,150 nodes in tree10. The explanation in for the increased number of visited nodes is similar as for gsr operation: the longer the word, the more subsets it can have, the more words in the tree, the more words with possible subsets there are.

In experiment3 we are interested about how a change in the alphabet size affects the operations. Five trees are created with maxSizeWord 50 and numTreeWord 50,000, alphabetSize is 20, 40, 60, 80 and 100, for tree1, tree2, tree3, tree4 and tree5, respectively. The number of nodes in the trees are: 869,373, 1,011,369, 1,069,615, 1,102,827 and 1,118,492. The test set contains 5,000 words.
When increasing alphabetSize the tree becomes sparser—the number of child nodes of a node is larger, but the number of nodes in all five trees is roughly the same. For gsr and more notably gsb operation, visit less nodes for the same input word length: the average number of visited nodes decreased when alphabetSize increases. The esr operation on the other hand visits more nodes in trees with larger alphabetSize. The number of visited nodes of esr increases with the increase of alphabetSize. This is because it is harder to find supersets of given words, when the number of symbols that make up words is larger. The effect is greater on word lengths below half maxSizeWord. The number of visited nodes starts decreasing rapidly after a certain word length. At this point the operation does not find any supersets and it returns false. gsr is not affected much by the change of alphabetSize. The greatest change happens when increasing alphabetSize over 20 (tree1). The number of visited nodes in trees 2 to 5 is almost the same, but it does decrease with every increase of alphabetSize.

In tree1 esb visits on average 3 nodes. When we increase alphabetSize the number of visited nodes also increases, but as in gsr the difference between trees 2 to 5 is small.

The change of alphabetSize has a greater effect on longer input words for the gsr operation. The number of visited nodes decreased when alphabetSize increased. Here again the biggest change is when going over alphabetSize 20. With every next increase, the difference in the number of visited nodes is smaller.
4. Related work

The initial implementation of SetTrie was in the context of a datamining tool fdep which is used for the induction of functional dependencies from relations. SetTrie serves there for storing and retrieving hypotheses that basically correspond to sets.

The data structure we propose is similar to trie [5]. Since we are not storing sequences but sets we can exploit the fact that the order in sets is not important. Therefore, we can take advantage of this to use syntactical order of elements of sets and obtain additional functionality of tries.

From the other perspective, our problem is similar to searching substrings in strings (for which Suffix trees can be used). The set of symbols of a substring is a subset of symbols of the string.

Baeza-Yates and Gonnet present an algorithm [1] for searching regular expressions using Patricia trees as the logical model for the index. They simulate a finite automata over a binary Particia tree of words. The result of a regular expression query is a superset or subset of the search parameter.

Charikar et. al. [2] present two algorithms to deal with a subset query problem. The purpose of their algorithms is similar to existsSuperSet operation. They extend their results to a more general problem of orthogonal range searching, and other problems. They propose a solution for “containment query problem” which is similar to our 2. query problem introduced in section 1.


5. Conclusions

The paper presents a data structure SetTrie that can be used for efficient storage and retrieval of subsets or supersets of a given word. The performance of SetTrie is shown to be good enough for manipulating sets of sets in practical applications.

Enumeration of subsets of a given universal set $U$ is very common in machine learning [3] algorithms that search hypotheses space ordered in a lattice. Often we have to see if a given set, a subset or a superset has already been considered by the algorithm. Such problems include discovery of association rules, functional dependencies as well as some forms of propositional logic.

Finally, the initial experiments have been done to investigate if SetTrie can be employed for searching substrings and superstrings in texts. For this purpose the data structure SetTrie has to be augmented with the references to the position of words in text. While the data structure is relatively large “index tree”, it may still be useful because of the efficient search.

References