Univerza na Primorskem
Fakulteta za matematiko, naravoslovje
in informacijske tehnologije
Koper, 29.08.2017.

NAME: STUDENT NUMBER:

_____   _____

SURNAME: SIGNATURE:

_____   _____

Algebra III - Abstraktna algebra

1. (a) Denote by S a set of size $n \in \mathbb{N}$. Explain (and compute the number) on how many of ways we can define binary operation on S! (30%)

(b) Show that all matrices of the following form

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}, \qquad \alpha \in \mathbb{R}$$

will form a group with respect to the (ordinary) matrix product. (70%)

2. We have the following permutations

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}, \quad \gamma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \quad \delta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}.$$

(a) Write all given permutations as product of disjoint cycles. (10%)

(b) Write all given permutations as a product of 2-cycles (that is as a product of transpositions). (10%)

(c) Compute $\alpha\beta$, $\beta\alpha$, $\alpha^{-1}\beta$, $\beta\alpha^2$, $(\alpha\beta)^{-2}$, $\alpha\beta\alpha^3$, $\gamma^{-1}\beta\gamma^{-1}$ and $\gamma\beta^2\alpha$. (80%)

3. (a) Find all left cosets of the subgroup $\langle 4 \rangle$ in group $\mathbb{Z}_{12}$ and write down Cayley table for $\mathbb{Z}_{12}/\langle 4 \rangle$. (40%)

(b) If there exists, find nontrivial homomorphism $\varphi : \mathbb{Z}_{12} \to \mathbb{Z}_4$. (60%)

4. Dihedral group $G = D_6$ acts in a natural way on regular 6-gon, which vertices we will denote by $X = \{1, 2, 3, 4, 5, 6\}$. Same action is true for subgroups $H = \langle \rho \rangle$, $K = \langle \tau \rangle$ and $L = \langle \tau\rho \rangle$ where $\rho = (123456)$ and $\tau = (26)(35)$.

(a) Find all orbits of actions of G, H, K and L on X!

(b) Recall: Action is transitive, if for any $x_1, x_2 \in X$, there exists $g \in G$ such that $x_1 = g * x_2$. Explain which actions of G, H, K and L are transitive!

(c) Recall: Action is faithful, if $\forall g \in G : g * x = x \; \forall x \in X \Rightarrow g = e$. Is action of G on X faithful? What about the other actions?

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".