Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
Koper, 13.06.2017.

NAME: STUDENT NUMBER:

SURNAME: SIGNATURE:

Algebra III - Abstraktna algebra

1. Let $S_{[0,1]}$ denote the set of all one-to-one mappings from interval $[0,1]$ onto interval $[0,1]$.

(a) Show that $S_{[0,1]}$ is a group under the composition of mappings. (10%)

(b) Consider $T = \{\alpha \in S_{[0,1]} \mid \alpha(0) = 0\}$. Show that T is a subgroup of $S_{[0,1]}$ (please explain also why T is nonempty). (30%)

(c) Show that $\{\sigma \in S_{[0,1]} \mid \sigma(0) = 1\}$ is a left coset of T in $S_{[0,1]}$. (40%)

(d) Find $[S_{[0,1]} : T]$. (20%)

2. Construct Cayley (multiplication) tables for alternating groups A_2 and A_3. Is it true that $\mathbb{Z}_3 \cong A_3$? (Carefully explain your answer!)

3. Let $\mathbb{R}^*$ denote group of nonzero real numbers with respect to (ordinary) multiplication. Use the first isomorphism theorem to prove that

$$\mathbb{R}^*/\langle -1 \rangle \cong \mathbb{R}^+.$$

4. Show that every abelian group of order 15 is cyclic.

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.