NAME: STUDENT NUMBER:

_____   _____

SURNAME: SIGNATURE:

_____   _____

Algebra III - Abstraktna algebra

1. Compute the number of elements of order 12 in $\mathbb{Z}_3 \times \mathbb{Z}_4 \times U(9)$.

2. By using the first isomorphism theorem, show that $\mathbb{Z} \times \mathbb{Z}/\langle(2,7)\rangle \cong \mathbb{Z}$.

3. Let 12345 denote regular pentagon, that is denote vertices of regular pentagon by numbers 1, 2, 3, 4 and 5. Let G be a group of all symmetries of regular pentagon 12345 and let $H = G_1$ (H is stibilizer of 1 in G).

(a) Find an element $\alpha \in G$ such that $\alpha(1) = 4$. What is the orbit of 1 under G.   (20%)

(b) Find an element $\beta \in H$ such that $\beta(2) = 5$. What is the orbit of 2 under H.   (20%)

(c) List all elements of H_2.   (30%)

(d) Use the orbit-stabilizer theorem to prove that $|G| = 10$.   (30%)

4. Find the possible number of Sylow 3-subgroups and Sylow 7-subgroups in a group of order 3087.

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".

(Ta stranica je namerno pustil prazna.)