



Univerza na Primorskem  
Fakulteta za matematiko, naravoslovje  
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NAME:

STUDENT NUMBER:

SURNAME:

SIGNATURE:

## Algebra III - Abstraktna algebra

1. (a) Let  $X$  be a nonempty set. Endow  $\mathcal{P}(X)$  with operation  $\setminus$  (difference)

$$\mathcal{A} \setminus \mathcal{B} := \{x \mid x \in \mathcal{A} \text{ and } x \notin \mathcal{B}\}, \quad \forall \mathcal{A}, \mathcal{B} \in \mathcal{P}(X).$$

Is the set  $\mathcal{P}(X)$  closed under operation  $\setminus$ ? Is operation  $\setminus$  associative on the set  $\mathcal{P}(X)$ ?  
Carefully explain your answer! (50%)

- (b) Let  $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 3 & 6 & 2 & 8 & 4 & 9 & 5 & 1 \end{pmatrix}$  denote a permutation. Find  $\pi^{-1}$ ,  $\pi^{2017}$  and  $\pi^{-2}\pi^4$ .

Write  $\pi^{-2}\pi^4$  as a product of 2-cycles (that is, as a product of transpositions). (50%)

2. Find all elements of order 9 in the group  $\mathbb{Z}_{108}$ . For every subgroup of order 9 in  $\mathbb{Z}_{108}$  find all possible generators.

3. (a) Let  $G = \{1, 8, 12, 14, 18, 21, 27, 31, 34, 38, 44, 47, 51, 53, 57, 64\}$  denote a group under multiplication modulo 65, and let  $H = \langle 12 \rangle$  be subgroup of  $G$ . Write down Cayley table for  $G/H$ . (70%)

- (b) Find the order of element  $8\langle 16 \rangle$  in the group  $U(105)/\langle 16 \rangle$ . (30%)

4. Let  $\mathcal{O}$  be the symmetry group of the cube (rotation, reflection, glide reflection,...).  $\mathcal{O}$  acts on the set  $\{v_1, v_2, \dots, v_8\}$  of vertices of a cube. Determine the stabilizer of a vertex  $v_1$  in the group  $\mathcal{O}$ . Use the orbit-stabilizer theorem to prove that  $|\mathcal{O}| = 48$ .

**Instructions:** Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".