Univerza na Primorskem
Fakulteta za matematiko, naravoslovje
in informacijske tehnologije
Koper, 04.06.2018.

NAME: ____________________ STUDENT NUMBER: ____________________

SURNAME: ____________________ SIGNATURE: ____________________

Algebra III - Abstract algebra

1. On set $\mathbb{Z}$ we have two binary operations $*$ and $\circ$, defined on the following way

$$a * b := a + 2b \qquad \text{and} \qquad a \circ b := 2ab.$$

(i) Check is it true that these two binary operation are commutative?

(ii) Check is it true that $(\mathbb{Z}, *)$ and $(\mathbb{Z}, \circ)$ are grups?

(iii) Check is it true that

$$x \circ (y * z) = (x \circ y) * (x \circ z) \qquad \text{and} \qquad x * (y \circ z) = (x * y) \circ (x * z)$$

hold for all $x, y, z \in \mathbb{Z}$.

2. Show that $\langle a, b \,|\, a^2 = b^3 = e,\ ab = ba \rangle \cong \mathbb{Z}_6$.

3. Let $H = \langle (9, 1) \rangle$ denote subgrup of a group $G = \mathbb{Z}_{18} \times \mathbb{Z}_2$.

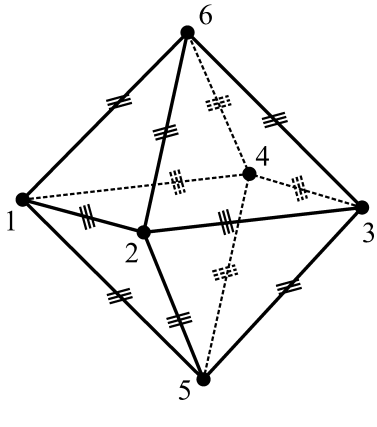
(a) Show that H is normal subgroup of a group G. (10%)

(b) Write all elements of quotient group $G/H = \mathbb{Z}_{18} \times \mathbb{Z}_2 / \langle (9, 1) \rangle$. (15%)

(c) Which known group is isomorphic with $\mathbb{Z}_{18} \times \mathbb{Z}_2 / \langle (9, 1) \rangle$. (60%)

(d) Find order of element $(2, 0) + H$ in a group G/H. (15%)

4. Let $\mathcal{O}$ be the symmetry group of the octahedron (rotation, reflection, glide reflection,...). $\mathcal{O}$ acts on the set $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ of vertices of a octahedron. Determine the stabilizer of a vertex v_1 for this action. Determine the stabilizer of a set $\{v_1, v_3\}$ for this action. Use the orbit-stabilizer theorem to calculate $|\mathcal{O}|$. Which known group is isomorphic to $\mathcal{O}$?



Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.