Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
Koper, 12.02.2018.

NAME: ________ STUDENT NUMBER: ________

SURNAME: ________ SIGNATURE: ________

Algebra III - Abstract algebra

1. (a) Let a and b denote two integers, and let $d = \gcd(a, b)$. If $H = \{am + bn \mid m, n \in \mathbb{Z}\}$ show that then

$$H = d\mathbb{Z}.$$

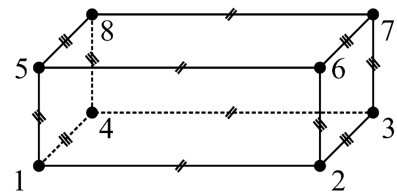
(40%)

(b) Find all subgroups of $(\mathbb{Z}, +)$. (60%)

2. (a) Show that groups $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$ are not isomorphic. (50%)

(b) Let S_7 be a given group. Find maximum order of elements from S_7 (largest order which some element from S_7 has). Find the number of elements of order 10 in S_7. (50%)

3. Let $\mathcal{O}$ be the symmetry group of the rectangular cuboid (rotation, reflection, glide reflection,...). $\mathcal{O}$ acts on the set $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ of vertices of a rectangular cuboid. Determine the stabilizer of a vertex v_1 for this action. Determine the stabilizer of a set $\{v_1, v_8\}$ for this action. Use the orbit-stabilizer theorem to calculate $|\mathcal{O}|$. Which known group is isomorphic to $\mathcal{O}$?



4. Let G denote simple group of order 1188. How many elements of order 11 are in G?

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.