Univerza na Primorskem
Fakulteta za matematiko, naravoslovje
in informacijske tehnologije
Koper, 22.01.2018.

NAME: ____________________ STUDENT NUMBER: ____________________

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Algebra III - Abstract algebra

1. (a) Let $G = \mathrm{Mat}_{2\times 2}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,\middle|\, a, b, c, d \in \mathbb{R} \right\}$ be a given group with respect to operation of addition (ordinary addition of matrices). Show that the set $H = \{A \in G \,|\, \mathrm{trace}(A) = 0\}$ is a subgroup of G. (60%)

(b) Let H denote nontrivial subgroup of G (that is $H \leq G$ and $H \neq G$). Show that $\langle G\backslash H \rangle = G$. (40%)

2. Let

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 6 & 7 & 5 & 1 & 3 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 3 & 5 & 1 & 2 & 6 & 4 \end{pmatrix}.$$

Find π^{-1} and π^{2018}.

3. (a) Let $G = (\mathrm{Mat}_{2\times 2}(\mathbb{Z}), +)$ and $H = \{A \in G \,|\, \mathrm{trace}(A) = 0\}$. Show that H is normal subgroup in G and show that $G/H \cong \mathbb{Z}$. (60%)

(b) Construct Cayley table for $\mathrm{Aut}(\mathbb{Z}_8)$. (40%)

4. Recall that center of a group G is defined as follows: $Z(G) = \{a \in G \,|\, ax = xa \text{ for } \forall x \in G\}$. Show that

$$Z(\mathrm{GL}_2(\mathbb{R})) = \mathbb{R} \cdot \mathrm{id}.$$

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.