Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
Koper, 12.01.2018.

NAME: ____________________ STUDENT NUMBER: ____________________

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Algebra III - Abstract algebra

1. Find the index of $\langle 3 \rangle$ in group $\mathbb{Z}_{24}$. Find the index of $\langle (2,3) \rangle$ in group $\mathbb{Z}_4 \times \mathbb{Z}_6$. Write down Cayley table for factor group $\mathbb{Z}_{24}/\langle 3 \rangle$.

2. (a) Let G denote a group of order 77 and let H be subgroup of G of order 11. Show that H is normal subgroup of G. (50%)

(Recall: For finite subgroups H and K of a group G we have

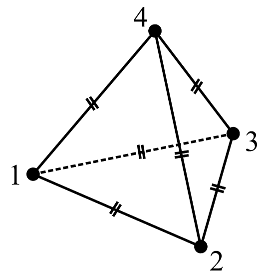
$$|HK| = \frac{|H||K|}{|H \cap K|}$$

where $HK = \{hk \,|\, h \in H,\, k \in K\}$.)

(b) Let H and K be normal subgroups of G, such that $H \cap K = \{e\}$, $|H| = p$ (p is prime number) and $|K| = 4$. If $HK = G$ show that then G is Abelian. (50%)

(Recall: If H and K are normal subgroups in G and if $H \cap K = \{e\}$ then $hk = kh$ for any $h \in H$ and $k \in K$).

3. Let $\mathcal{O}$ be the symmetry group of the tetrahedron (rotation, reflection, glide reflection,...). $\mathcal{O}$ acts on the set$\{v_1, v_2, v_3, v_4\}$ of vertices of a tetrahedron. Determine the stabilizer of a vertex v_1 for this action.. Use the orbit-stabilizer theorem to calculate $|\mathcal{O}|$.



4. Let G denote simple group of order 168. How many elements of order 7 are in G?

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".