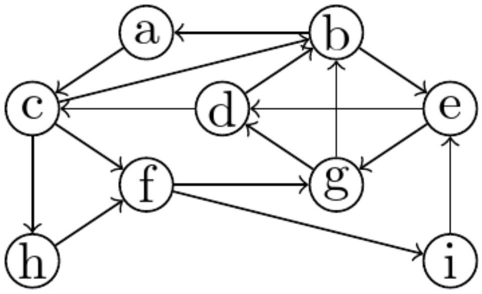
20 Eulerian and Hamiltonian Graph.

102. Does the following digraph have



(i) an Eulerian circuit?

(ii) a non-closed Eulerian trail?

(iii) a Hamiltonian cycle?

(iv) a Hamiltonian path?

For each question, if the answer is affirmative, draw the circuit/trail/cycle/path. If the answer is negative, provide a justification.

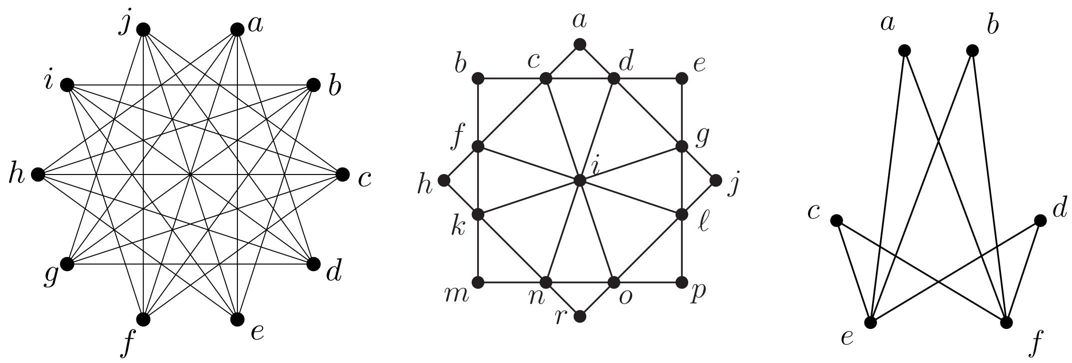
103. For each of the following graphs, determine:

(a) whether it contains an induced subgraph that is non-bipartite and has 6 vertices;

(b) whether it is Eulerian;

(c) whether it contains a non-closed Eulerian trail;

(d) whether it is Hamiltonian.

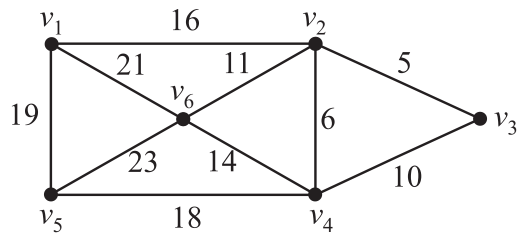


G_1 G_2 G_3

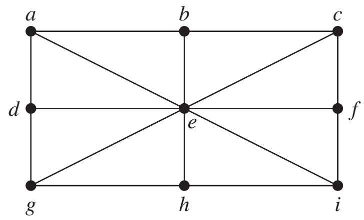
104. Let G be a simple graph with n vertices and m edges, where $n \geq 3$. Assume that $m \geq \frac{(n-1)(n-2)}{2} + 2$. Prove that G is Hamiltonian. Does the converse hold?

21 Eulerian Graph. Hamiltonian Graph. Kruskal's Algorithm.

105. Use Kruskal's algorithm to find a minimum spanning tree for the graph given in the figure below. Describe all the steps of the algorithm and state the weight of the resulting tree.



106. Given the following graph G:



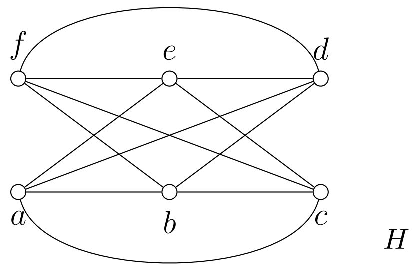
(a) Determine if G has a Hamiltonian cycle. If G has a Hamiltonian cycle, find one. If it does not, provide a justification explaining why such a cycle does not exist.

(b) Is G an Eulerian graph? Justify your answer.

(c) Provide an example of a spanning tree for the graph G shown.

(d) Give an example of a spanning subgraph of G that is not bipartite.

107. Consider the following graph H:



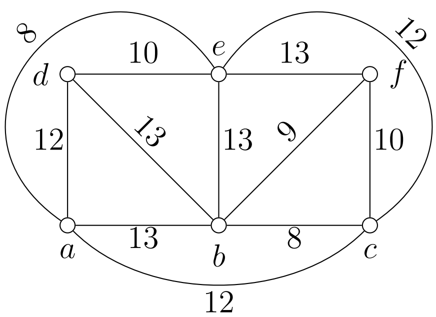
(a) Is H regular?

(b) Is H Hamiltonian?

(c) Is H Eulerian?

Justify your answers.

108. Consider the following graph G with edge weights:



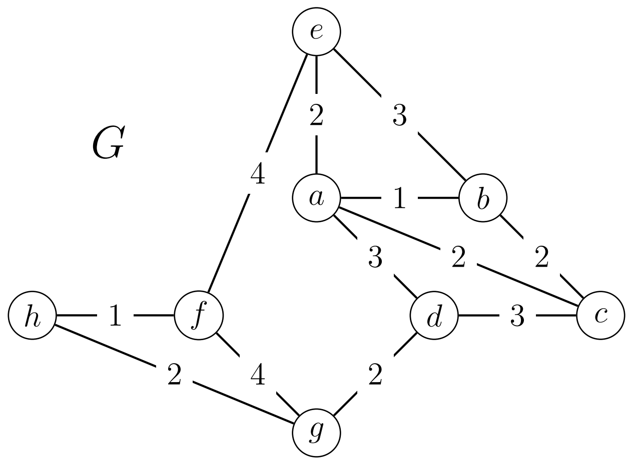
(a) Use Kruskal's algorithm to find the minimum spanning tree in graph G and determine its weight. Describe each step of the algorithm.

(b) Is the minimum spanning tree uniquely determined? Justify your answer.

(c) Can we orient the edges of graph G such that the resulting digraph D is strongly connected? Provide an appropriate orientation or justify why it does not exist.

(d) Can we orient the edges of graph G such that the resulting digraph D contains an Eulerian circuit? Provide an appropriate orientation or justify why it does not exist.

109. Let G be the following weighted connected graph.



(a) Use Kruskal's greedy method to find the minimum spanning tree (minimum connector) of graph G. Explain all the steps required to obtain your solution.

(b) Determine if the tree computed in the previous part is unique, i.e., if there is another tree with the same weight that can be found using Kruskal's method.

(c) Is G Hamiltonian? If so, find a Hamiltonian cycle in G.

(d) Calculate $\chi(G)$.

(e) Show that there is a graph homomorphism from G to the complete graph K_r, where $r = \chi(G)$.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.