

FINITE GEOMETRIES

Sixth Irsee Conference

28 August - 3 September 2022
Irsee, Germany

Organisers: Ilaria Cardinali - Michel Lavrauw - Klaus Metsch - Alexander Pott



1 INFORMATION

The Sixth Irsee Conference on Finite Geometries extends a series of previous meetings which took place at the Isle of Thorns (2000), in Oberwolfach (2001), and in Irsee (2003, 2006, 2011, 2014 and 2017).

CONFERENCE TOPICS

Combinatorial structures in Galois geometries; Finite Incidence Geometry; Algebraic curves and varieties over finite fields; Geometric and algebraic coding theory; Finite groups and geometries; Algebraic design theory.

MAIN SPEAKERS

Elisa Gorla (Université de Neuchâtel, Switzerland)

Anna-Lena Horlemann (University of St.Gallen, Switzerland)

Gábor Korchmáros (University of Basilicata, Italy, and Eötvös L. University of Budapest, Hungary,
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Giuseppe Mazzuoccolo (University of Verona, Italy)

John Sheekey (University College Dublin, Ireland)

Geertrui Van de Voorde (University of Canterbury, New Zealand)

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2 SCHEDULE

	Monday	Tuesday	Wednesday	Thursday	Friday
09:30-10:30	Korchmáros	Sheekey	Gorla	Mazzuoccolo	Van de Voorde
10:30-11:00	BREAK	BREAK	BREAK	BREAK	BREAK
11:00-11:25	Weiner	Pinero		Schmidt	Jameson
11:30-11:55	Sziklai	Kiermaier		Ernst	Ihringer
12:00-12:25	Kurz	De Bruyn		Weiß	Pavese
12:30-12:55				Zini	Zanella
11:00-12:00			Horlemann		
12:10-12:35			Kılıç		
12:40-13:05			Giuzzi		
13:00	LUNCH	LUNCH		LUNCH	LUNCH
13:15			LUNCH		
14:30-14:55	Berardini	Storme		Santonastaso	De Beule
15:00-15:25	Nagy	Abdukhalikov		D'haeseleer	Gavrilyuk
15:30-15:55	Kölsch	Zullo		Landjev	Mannaert
16:00-16:30	BREAK	BREAK		BREAK	BREAK
16:30-16:55	Botteldoorn	Adriaensen		Hawtin	Ravagnani
17:00-17:25	Csajbók	Rousseva		Jurrius	Mattheus
17:30-17:55	Neri	Ball		Doyen	Lavrauw
18:00	RECEPTION				
18:30		DINNER			
19:00	DINNER		DINNER	DINNER	CONFERENCE DINNER
20:00		CONCERT			

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3 INVITED ABSTRACTS

Generalized weights of convolutional codes

Elisa Gorla

University of Neuchâtel

(Joint work with Flavio Salizzoni)

In 1997 Rosenthal and York define generalized Hamming weights for convolutional codes, by regarding a convolutional code as an infinite dimensional linear code endowed with the Hamming metric. In this talk, we will propose a new definition of generalized weights of convolutional codes, that takes into account the underlying module structure of the code. We will derive the basic properties of our generalized weights and discuss the relation with the previous definition. We will establish upper bounds on the weight hierarchy of MDS and MDP codes and show that that, depending on the code parameters, some or all of the generalized weights of MDS codes are determined by the length, rank, and internal degree of the code. If time allows, we will also define optimal anticodes and discuss their basic properties.

The Density of Optimal Error-Correcting Codes in Various Metric Spaces

Anna-Lena Horlemann

University of St.Gallen, Switzerland

(Joint work with Anina Gruica, Alberto Ravagnani and Nadja Willenborg)

The density of good error-correcting codes has always been of interest in information theory, classically motivated by the question how good a random code will perform in terms of information rate and error correction capability. More recently, densities of codes have also become important in the field of code-based cryptography, where random (linear) codes are chosen in the design of a digital signature scheme. It has long been known that linear MDS codes are dense for growing field size and sparse (or rather non-existent) for growing length of the code. Similar results were obtained a few years ago for linear optimal rank-metric (MRD) codes. Surprisingly though, shortly after it was shown that sublinear MRD codes, i.e., those MRD codes that are linear over a subfield of the ambient field, are not dense for growing field size. Hence, the question arose if sublinear (or additive) MDS codes are sparse or dense for growing field size, and if similar results can be obtained for other metric spaces. In this talk we show a generalized framework for determining the densities of non-linear, sublinear and linear optimal codes in various metric spaces. We then apply these techniques to codes in the Hamming, rank, sum-rank, Lee and subspace metric. Moreover, we will describe the relationship to finite geometry, e.g. to n -arcs (of points) and n -arcs of higher dimensional projective spaces.

Plane algebraic curves with many symmetries, and complete (k, n) -arcs

Gábor Korchmáros

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Eötvös L. University of Budapest (Hungary) AC Research Group

(Joint work with H. Borges, G.P.Nagy, P. Speziali and T. Szőnyi)

Let $PGL(3, q)$ be the 3-dimensional projective linear group defined over a finite field \mathbb{F}_q and viewed as a subgroup of $PGL(3, \mathbb{K})$, \mathbb{K} being an algebraic closure of \mathbb{F}_q . For the seven maximal (non sporadic) subgroups G of $PGL(3, q)$, we consider the G -invariant (projective, irreducible) plane curves of $PG(2, q^m)$ where $m \geq 1$. In [3], for each such group G we computed the minimum degree $d(G)$ of G -invariant curves, provided a classification of all G -invariant curves of degree $d(G)$, and determined the first gap in the spectrum of the degrees of all G -invariant curves. We also pointed out that G -invariant curves of degree $d(G)$ happen to have particular geometric features such as Frobenius non-classicality and an unusual variation of the number of \mathbb{F}_{q^i} -rational points. It seems conceivable that they also may have several interesting combinatorial properties. A degree n plane algebraic curve \mathcal{C}^n with k points in $PG(2, q^m)$ often defines a (k, n) -arc in $PG(2, q^m)$, and a well known problem raised in [6], is to find plane curves for which the arising (k, n) -arc is complete; see also [2, 4]. We single out two cases:

- (i) the $(k, q + 1)$ -arc in $PG(2, q^m)$, $m = 2s$ even, arising from the Hermitian curve \mathcal{H}_q of degree $q + 1$ (and left invariant by $G \cong PGU(3, q)$). Here, $k = q^{2s} \pm q^{s+1}(q - 1) + 1$ according as s is odd or even.
- (ii) the $(k, q + 1)$ -arc in $PG(2, q^m)$ arising from a rational curve Γ_q of degree $q + 1$ with q odd (and left invariant by $G \cong PGL(2, q)$). Here, $k = q^m + 1$.

In both cases, $k \approx q^m$ and the $(k, q + 1)$ -arc of $PG(2, q^m)$ is complete, apart from just one possibility for the Hermitian case for $s = 3$ where the completeness problem is open.

To deal with the completeness problem, we adopt a natural algebraic approach already used by D. Bartoli and G. Micheli [5] (depending on a previous work by Guralnick, Tucker and Zieve [7]). The essential idea is to express the condition that a point $P \in PG(2, q^m)$ is incident with a line which meets \mathcal{C}^n in n pairwise distinct points of $PG(2, q^m)$, in terms of the Galois closure of the algebraic extension $F|F_P$ where F is the function field of \mathcal{C}^n and F_P is the rational subfield of F arising from the projection of \mathcal{C} from P . It should be stressed that in almost all cases (including those studied in [5]), if $P \notin \mathcal{C}^n$ and $P \notin PG(2, q^2)$ then the Galois group of the Galois closure of $F|F_P$ is the symmetric or the alternating group. Instead, for each of the two above two curves, the Galois group is much smaller as it is isomorphic to $PGL(2, q)$ acting naturally on the roots of the polynomial associated with $F|F_P$. To prove this, we also rely on previous work of Abhyankar [1] and van der Waerden [8].

References

- [1] S.S. Abhyankar, Galois theory on the line in nonzero characteristic. *Bull. Amer. Math. Soc. (N.S.)* **27** (1992), 68–133.
- [2] D. Bartoli, M. Giulietti, G. Zini, Complete $(k, 3)$ -arcs from quartic curves, *Des. Codes Cryptogr.* **79** (2016), 487–505.
- [3] H. Borges, G. Korchmáros, P. Speziali, Plane curves with a large linear automorphism group in characteristic p , arXiv:2202.05765 [math.AG].
- [4] H. Borges, B. Motta, and F. Torres, Complete arcs arising from a generalization of the Hermitian curve. *Acta Arith.* **164** (2014), 101–118.

- [5] D. Bartoli and G. Micheli, Algebraic constructions of complete m -arcs, *Combinatorica* (2022).
<https://doi.org/10.1007/s00493-021-4712-5>.
- [6] M. Giulietti, F. Pambianco, F. Torres, and E. Ughi, On complete arcs arising from plane curves, *Des. Codes Cryptogr.* **25** (2002), 237–246.
- [7] R. Guralnick, T.J. Tucker, M.E. Zieve, Exceptional covers and bijections on rational points, *Int. Math. Res. Not. IMRN* 2007, no. 1, Art. ID rnm004, 20 pp.
- [8] B.L. van der Waerden, Die Zerlegungs- und Trägheitsgruppe als Permutationsgruppen, *Math. Annalen* **111** (1935), 731-733.

A geometric approach to determine an optimal 2-dimensional flow on a graph

Giuseppe Mazzuoccolo

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(Joint work with D.Mattiolo, J.Rajník and G.Tabarelli)

The theory of integer nowhere-zero flows on finite graphs represents a very active research area in graph theory. The generalization to real numbers is also well-studied, while very few is known in the complex case or, more in general, for flows taking values in \mathbb{R}^d . We define a d -dimensional nowhere-zero r -flow on a graph G , (r, d) -NZF from now on, as a nowhere-zero flow such that the flow value assigned to each edge is an element of \mathbb{R}^d whose (Euclidean) norm lies in the interval $[1, r - 1]$. In this talk, we mainly consider the parameter $\phi_d(G)$, which is the minimum of the real numbers r such that G admits a (r, d) -NZF. For every bridgeless graph G , the 5-flow Conjecture claims that $\phi_1(G) \leq 5$, while a conjecture by Jain suggests that $\phi_d(G) = 2$, for all $d \geq 3$. Here, we address the problem of finding a possible upper-bound in the case $d = 2$ and we discuss some connections between this problem and some other well-known conjectures. Finally, we propose a geometric method to describe a $(r, 2)$ -NZF of a cubic graph in a compact way, and we apply it in some instances. In general, the exact computation of $\phi_2(G)$ for an arbitrary graph G seems to be an hard task even in very small and symmetric cases. In particular, the exact value is determined only for graphs belonging to special families where a lower bound can be trivially proved. By using a combination of geometric and combinatorial arguments we compute a lower-bound for the 2-dimensional flow number of a cubic graph in terms of its odd-girth.

References

- [1] D.Mattiolo,G.Mazzuoccolo,J.Rajník and G.Tabarelli *On d -dimensional nowhere-zero r -flows on a graph*, submitted for the publication.
- [2] C.Thomassen, *Group flow, complex flow, unit vector flow, and the $(2 + \epsilon)$ -flow conjecture*, J. Comb. Theory B 108 (2014), 81–91.

Semifields and MRD Codes: Invariants of Interest

John Sheekey

University College Dublin

Finite semifields, which are division algebras in which multiplication is not assumed to be associative, have been studied for over a century for a variety of reasons [6]. Dickson [4] initiated the study of finite semifields in 1905, constructing the first nontrivial examples and thus showing that the Wedderburn-Dickson theorem (that all finite associative division algebras are equivalent to fields) cannot be extended.

Semifields correspond to various interesting geometrical structures with special symmetry properties. They give rise to special types of *projective planes*, as studied by for example Albert [1] and Knuth [5], and *spreads*, as studied by André [2] and Bruck-Bose [3]. In more recent years, connections to topics such as *linear sets* [8], *tensors* [7], and *MRD codes* [9] have lead to renewed focus.

Each of these settings has its own advantages, particularly in providing useful and interesting invariants, which can be used to distinguish semifields from each other, and to understand their structure. In this talk we will survey these invariants, from classical objects such as the *nuclei*, to more recent concepts such as *geometric invariants* arising from linear sets, the *tensor rank*, the *BEL rank*, and the *covering radius*.

References

- [1] Albert, A. A.: *Generalized twisted fields*, Pacific J. Math. 11 (1961), 1-8.
- [2] André, J.: *Über nicht-Desarguessche Ebenen mit transitiver Translationsgruppe* Math. Z. 60 (1954), 156–186.
- [3] Bruck, R.H., Bose, R.H.; *The construction of translation planes from projective spaces*, J. Algebra 1 (1964), 85–102.
- [4] Dickson, L. E.; *On finite algebras*, Nachrichten der Gesellschaften der Wissenschaften zu Göttingen (1905) 358-393.
- [5] Knuth, D. E.; *Finite semifields and projective planes*, J. Algebra 2 (1965) 182-217.
- [6] Lavrauw, M.; Polverino, O.: Finite Semifields. Chapter in *Current research topics in Galois geometries*. Nova Academic Publishers (J. De Beule and L. Storme, Eds.).
- [7] Lavrauw, M.; *Finite semifields and nonsingular tensors*, Des. Codes Cryptogr. 68 (2013), 205–227
- [8] Marino G., Polverino O., Trombetti R.; *On Fq -linear sets of $PG(3, q^3)$ and semifields*, J. Comb. Theory Ser. A 114 (2007) 769–788 .
- [9] Sheekey. J.; MRD Codes; Constructions and Connections, in K.-U. Schmidt and A. Winterhof (eds.), *Combinatorics and finite fields: Difference sets, polynomials, pseudorandomness and applications*. Radon Series on Computational and Applied Mathematics, 255–286. De Gruyter, 2019.

Quasi-polar spaces

Geertrui Van de Voorde

University of Canterbury, New Zealand

The study of point sets with few intersection numbers is at the core of finite geometry. Segre's famous characterisation of ovals in Desarguesian planes of odd order as conics only used the size of the point set and its intersection numbers with respect to lines. In higher dimensions, the same problem leads to the concept of *quasi-quadrics*, or more generally, *quasi-polar spaces*; a set of points with the same size and intersection numbers with respect to hyperplanes as a classical polar space. Using a technique called *pivoting*, De Clerck, Hamilton, O'Keefe and Penttila constructed quasi-quadrics that are not quadrics [2].

In the first part of this talk, we will discuss pivoting in detail and present some of the related recent results of [4].

The second part of this talk deals with *unitals*, which are quasi-polar spaces in a Desarguesian plane: they have the same intersection numbers as the Hermitian curve. We will review some of the classical results about unitals, and present new results on Buekenhout-Tits unitals [3] in the same spirit as the results on Buekenhout-Metz unitals of [1].

References

- [1] N. Abarzúa, R. Pomareda, and O. Vega. Feet in orthogonal-Buekenhout–Metz unitals. *Adv. Geom.* **18 (2)** (2018), 229–236.
- [2] F. De Clerck, N. Hamilton, C. O'Keefe and T. Penttila. Quasi-quadrics and related structures. *Australas. J. Combin.* **22** (2000), 151–166.
- [3] J. Faulkner and G. Van de Voorde. On the Equivalence, Stabilisers, and Feet of Buekenhout-Tits Unitals. In preparation.
- [4] J. Schillewaert and G. Van de Voorde. Quasi-polar spaces. *ArXiv 2109.12710*.

4 CONTRIBUTED ABSTRACTS

Linear codes from arcs and quadrics

Kanat Abdukhalikov

UAE University

(Joint work with Duy Ho)

Using characterizations of ovals, arcs and elliptic quadrics recently described in polar coordinates, we construct some families of LCD, self-orthogonal, three-weight and four-weight linear codes. We also present characterizations of some cyclic codes with specific parameters. Finally, we give simple presentations of Denniston maximal arcs in $PG(2, q)$ and elliptic quadrics in $PG(3, q)$.

On additive MDS codes with linear projections

Sam Adriaensen

Vrije Universiteit Brussel

(Joint work with Simeon Ball)

In this talk, we will discuss additive MDS codes over finite fields. We will be working in the following framework. Let q be a prime power, and let h , k , and $n > k$ be positive integers. We are interested in subsets $C \subset \mathbb{F}_{q^h}^n$ such that:

1. C is a hk -dimensional vector space over \mathbb{F}_q ,
2. the minimum Hamming distance of C is $n - k + 1$, i.e. given two distinct vectors in C , they have different entries in at least $n - k + 1$ positions.

Note that the first condition is equivalent with C being an additive code if q is prime, and with C being a linear code if $h = 1$.

It is generally believed that the only long MDS codes are the extended Reed-Solomon codes, with some known exceptions. This belief is often referred to as the MDS conjecture. There has been significant progress in verifying the MDS conjecture, mainly in the context of linear codes. In this talk, we will describe the geometric framework to work with additive MDS codes over finite fields, and support some evidence that long additive MDS codes are linear.

Stabiliser codes and quantum sets of lines

Simeon Ball

Universitat Politècnica Catalunya

We present a geometric framework for constructing additive and non-additive stabiliser codes which encompasses stabiliser codes and graphical non-additive stabiliser codes. This builds on work of Glynn et al. [3] who proved that a qubit stabiliser code with parameters $((n, 2^k, d))_2$ is equivalent to a set \mathcal{X} of lines in $PG(n - k - 1, 2)$ with the property that every co-dimension two subspace is skew to an even number of the lines of \mathcal{X} . And where d is at least the minimum number of dependent points on distinct lines of \mathcal{X} . This geometrical interpretation can be used to prove the non-existence of certain qubit stabiliser codes, see for example [2].

The generalisation here covers direct sums of stabiliser codes, as well as qupit codes, i.e. subspaces of $(\mathbb{C}^p)^{\otimes n}$, for all primes p .

Let $A = (a_{ij})$ be a symmetric $n \times n$ matrix with elements from \mathbb{F}_p with zeros on the diagonal, so the adjacency matrix of a simple weighted graph. We define a graphical set of lines as the set of n lines $\mathcal{X} = \{\ell_1, \dots, \ell_n\}$ of $PG(n - 1, 2)$, where

$$\ell_i = \langle e_i, a_i \rangle,$$

where e_i is the i -th vector in the canonical basis and a_i is the i -th column of A .

The main result of this talk is the following theorem, from [1].

Theorem 1 *A direct sum of cosets of a stabiliser code (which can simply be a stabiliser code) with parameters $((n, |T|(p - 1) + 1, d))_p$ can be obtained from a graphical set of n lines \mathcal{X} of $PG(n - 1, 2)$ and a set of points T with the property that no two distinct points of T span a point which is the sum of $d - 1$ or less points on distinct lines of \mathcal{X} .*

References

- [1] S. Ball and P. Puig, The geometry of non-additive stabiliser codes, [arXiv:2107.11281](https://arxiv.org/abs/2107.11281).
- [2] J. Bierbrauer, Stefano Marcugini, Fernanda Pambianco The non-existence of a $[[13, 5, 4]]$ quantum stabilizer code, *IEEE Transactions on Information Theory*, **57** (2011) 4788–4793.
- [3] D. G. Glynn, T. A. Gulliver, J. G. Maks and M. K. Gupta, *The Geometry of Additive Quantum Codes*, unpublished manuscript. (available online at <https://www.academia.edu/17980449/>)

On the number of rational points of curves over a surface in \mathbb{P}^3

Elena Berardini

Eindhoven University of Technology

(Joint work with Jade Nardi)

The number of rational points on a smooth projective absolutely irreducible curve C of genus g defined over the finite field \mathbb{F}_q is bounded by the famous Serre–Weil bound, namely $\#C(\mathbb{F}_q) \leq q + 1 + g[2\sqrt{q}]$. Several works have been devoted to improve this bound for a range of parameters, and to extend it to more general curves, possibly reducible or singular [4, 1, 3].

In this talk, we will show that the number of rational points on an irreducible curve of degree δ defined over a finite field \mathbb{F}_q lying on a surface S in \mathbb{P}^3 of degree d is, under certain conditions, bounded by $\delta(d + q - 1)/2$. Within a certain range of δ and q , this result improves all other known bounds in the context of space curves. The method we used is inspired by techniques developed by Stöhr and Voloch [4]. In their seminal work of 1986, they introduced the Frobenius orders of a projective curve and used them to give an upper bound on the number of rational points of the curve. After recalling some general results on the theory of orders of a space curve, we will study the arithmetic properties of curves lying on a surface in \mathbb{P}^3 , to finally prove the bound.

The talk is based on the preprint [2].

References

- [1] Yves Aubry and Marc Perret. *A Weil theorem for singular curves*. pages 1–8. De Gruyter, 1993.
- [2] Elena Berardini and Jade Nardi. *Curves on Frobenius classical surfaces in \mathbb{P}^3 over finite fields*. arXiv preprint arXiv:2111.09578, 2021.
- [3] Masaaki Homma. *A bound on the number of points of a curve in a projective space over a finite field*. Theory and applications of finite fields, volume 579 of Contemp. Math., pages 103 – 110. Amer. Math. Soc., Providence, RI, 2012.
- [4] Karl-Otto Stöhr and José Felipe Voloch. *Weierstrass Points and Curves Over Finite Fields*. Proceedings of The London Mathematical Society, 1, 1–19, 1986.

Minimal blocking sets in small Desarguesian projective planes

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(Joint work with Kris Coolsaet and Veerle Fack)

All minimal blocking sets (up to equivalence) in Desarguesian projective planes of order ≤ 9 were generated by computer. These blocking sets were then classified according to size of the set, and order of the projective automorphism group and collineation group. Explicit descriptions or constructions are given for some sets, in particular (but not exclusively) for those blocking sets with a fairly large automorphism group. Some of these constructions can be generalised to Desarguesian projective planes of higher order.

We have found $\text{PG}(2, 7)$ to have 1433 inequivalent minimal blocking sets; $\text{PG}(2, 8)$ has over 45 thousand and $\text{PG}(2, 9)$ has over 15 million minimal blocking sets (inequivalent under $\text{P}\Gamma\text{L}(3, 8)$ and $\text{P}\Gamma\text{L}(3, 9)$, respectively). We have been able to describe several of these blocking sets using unions of orbits of powers of Singer cycles, orbits of $\text{Sym}(4)$ and $\text{Sym}(5)$, sum-free sets, the Hessian configuration, algebraic curves, unitals, unions of Fano subplanes, . . . For the planes of order ≤ 8 , these results can be found in [1].

References

- [1] Coolsaet, K., Botteldoorn, A., and Fack, V., Classification of minimal blocking sets in small Desarguesian projective planes, *J. Combin. Des.* (2022), 30, 561– 580. <https://doi.org/10.1002/jcd.21842>

Notes on multiple blocking sets of $\text{PG}(2, q)$

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Put $q = p^n$, where p is a prime. A (t -fold) blocking set of $\pi \cong \text{PG}(2, q)$ is a point set meeting each line of π (in at least t -points). A t -fold blocking set is called minimal, if after removing any of its points, the remaining point set is not a t -fold blocking set. Let V denote the 3-dimensional \mathbb{F}_q -vector space whose subspace lattice defines π . A blocking set is called “linear” if it is the set of points defined by the non-zero vectors of an \mathbb{F}_p -subspace of V .

When $n > 1$, then the smallest known minimal blocking sets are linear. When n is even, then $\text{PG}(2, q)$ is the disjoint union of $q - \sqrt{q} + 1$ Baer-subplanes. The union of any t of them is a t -fold blocking set. When $n > 1$ is odd, then one can find two disjoint linear blocking sets in π , see [1, 2, 3], and the smallest known 2-fold blocking sets are obtained as their union.

In this talk I will explore the possibility of finding three disjoint linear blocking sets in order to construct the smallest known 3-fold blocking sets when $n > 1$ is odd.

References

- [1] G. BACSÓ, T. HÉGER, T. SZÓNYI: The 2-Blocking Number and the Upper Chromatic Number of $\text{PG}(2, q)$. *J. Combin. Des.*, **21(12)** (2013), 585–602.
- [2] D. BARTOLI, A. COSSIDENTE, G. MARINO, F. PAVESE: On cutting blocking sets and their codes. *Forum Math.*, **34(2)** (2022), 347–368.
- [3] J. DE BEULE, T. HÉGER, T. SZÓNYI, G. VAN DE VOORDER: Blocking and double blocking sets in finite planes. *Electron. J. Combin.*, **23(2)** (2016), #P2.5

A modular equality for Cameron-Liebler line classes in projective and affine spaces of odd dimension

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(Joint work with Jonathan Mannaert)

Cameron-Liebler line classes were introduced in [1] as orbits of irreducible collineation groups of $\text{PG}(d, q)$ having equally many point orbits as line orbits. When $\text{PG}(d, q)$ allows line spreads, i.e. if and only if d is odd, such a line set \mathcal{L} has the property that for any line spread \mathcal{S} , $|\mathcal{L} \cap \mathcal{S}| = x$ for some fixed natural number x , only dependent on \mathcal{L} .

It will be briefly illustrated that examples of non-trivial Cameron-Liebler line classes in $\text{PG}(3, q)$ are rare. This motivates older and more recent *non-existence* results, i.e. results that exclude values for the parameter x . One of the most consequential non-existence results is the following theorem.

Theorem 1 ([3, Theorem 1.1]) *Suppose that \mathcal{L} is a Cameron-Liebler line class with parameter x of $\text{PG}(3, q)$. Then for every plane and every point of $\text{PG}(3, q)$,*

$$\binom{x}{2} + m(m - x) \equiv 0 \pmod{q + 1}, \quad (1)$$

where m is the number of lines of \mathcal{L} in the plane, respectively through the point.

In this talk we present the following generalization.

Theorem 2 ([2]) *Let \mathcal{L} be a Cameron-Liebler line class with parameter x in $\text{PG}(n, q)$, with $n \geq 7$ odd. Then for any point p ,*

$$x(x - 1) + 2\bar{m}(\bar{m} - x) \equiv 0 \pmod{q + 1},$$

where \bar{m} is the number of lines of \mathcal{L} through p .

We will also discuss the affine version of both theorems.

References

- [1] P. J. Cameron and R. A. Liebler. Tactical decompositions and orbits of projective groups. *Linear Algebra Appl.*, 46:91–102, 1982.
- [2] J. De Beule and J. Mannaert. A modular equality for Cameron-Liebler line classes in projective and affine spaces of odd dimension. *Finite Fields Appl.*, 82:Paper No. 102047, 14, 2022.
- [3] A. L. Gavriluyk and K. Metsch. A modular equality for Cameron-Liebler line classes. *J. Combin. Theory Ser. A*, 127:224–242, 2014.

Quadratic sets on the Klein quadric

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Let $Q^+(5, q)$ be the Klein quadric in $\text{PG}(5, q)$. A set of points of $Q^+(5, q)$ is called a *quadratic set* if every plane of $Q^+(5, q)$ intersects it in a possibly degenerate quadric. There are thus five possible plane intersections (singleton, line, irreducible conic, two lines, whole plane), and we call the quadratic set *good* if at most two of these possibilities occur. We discuss classification and (non-)existence results for good quadratic sets, as well as some applications of them to open problems involving certain line sets in $\text{PG}(3, q)$ and hyperovals in $Q^+(5, q)$.

References

- [1] B. De Bruyn and M. Gao. Pseudo-embeddings and quadratic sets of quadrics. *Des. Codes Cryptogr.* 90 (2022), no. 1, 199–213.
- [2] B. De Bruyn. Quadratic sets on the Klein quadric. *J. Combin. Theory Ser. A* 190 (2022), Paper No. 105635, 49 pp.
- [3] B. De Bruyn, P. Pradhan, B. Sahu and B. K. Sahoo. A characterization of the family of secant lines to a hyperbolic quadric in $\text{PG}(3, q)$, q odd, Part II. *Discrete Mathematics*, to appear.

The André/Bruck-Bose representation of linear sets on a projective line

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(Joint work with Lins Denaux and Geertrui Van de Voorde)

We consider the André/Bruck-Bose representation of the projective plane $\text{PG}(2, q^t)$ in $\text{PG}(2t, q)$. We investigate the representation of a linear set of rank k on a line, different from the line at infinity in $\text{PG}(2, q^t)$. More precisely, we characterize the representation of tangent scattered linear sets on a line for $k = 3, t = 3$, tangent clubs with head point contained in the line at infinity for $k \geq 3, t \geq k$, and tangent clubs with head point not contained in the line at infinity for $k = 3, t \geq 3$. This characterisation of the André/Bruck-Bose representation of linear sets can be used to find optimal higgledy-piggledy sets.

Steiner triple systems with a given automorphism group

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(Joint work with William Kantor)

We investigate the following problem: Given a finite abstract group G , for which integers v is there a Steiner triple system of order v (i.e. a $2 - (v, 3, 1)$ design) whose full automorphism group is isomorphic to G ? The result is rather surprising. Some generalizations and open problems will be briefly discussed.

Intersecting theorems for finite general linear groups

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(Joint work with Kai-Uwe Schmidt)

A subset Y of the symmetric group \mathcal{S}_n is *t-intersecting* if $x^{-1}y$ fixes t elements in $[n]$ for all $x, y \in Y$ and it is *t-set-intersecting* if $x^{-1}y$ fixes a t -set of $[n]$ for all $x, y \in Y$. Deza and Frankl conjectured [1] and Ellis, Friedgut, and Pilpel proved [3] that the size of a t -intersecting set in \mathcal{S}_n is at most $(n-t)!$ for n sufficiently large compared to t . Moreover equality holds if and only if the t -intersecting set is a coset of the stabiliser of a t -tuple. Ellis proved [2] that the size of a t -set-intersecting set in \mathcal{S}_n is at most $t!(n-t)!$ for n sufficiently large compared to t and equality holds if and only if the t -set-intersecting set is a coset of the stabilizer of a t -set.

In this talk we discuss q -analogs of these results. We define a subset Y of $\text{GL}(n, q)$ to be *t-intersecting* if $x^{-1}y$ fixes a t -dimensional subspace of \mathbb{F}_q^n pointwise for all $x, y \in Y$. Whereas we define Y to be *t-space-intersecting* if $x^{-1}y$ fixes a t -dimensional subspace of \mathbb{F}_q^n for all $x, y \in Y$. It is shown that the size of a t -intersecting subset of $\text{GL}(n, q)$ is at most

$$[n-t]_q! \frac{(q-1)^n q^{\binom{n}{2}}}{(q-1)^t q^{\binom{t}{2}}}$$

for n sufficiently large compared to t . In addition it is shown that the size of a t -space-intersecting subset of $\text{GL}(n, q)$ is at most

$$[t]_q! [n-t]_q! (q-1)^n q^{\binom{n}{2}}$$

for n sufficiently large compared to t . Moreover we give a characterisation of the cases for which equality holds.

References

- [1] M. Deza, P. Frankl, *On the maximum number of permutations with given maximal or minimal distance*, J. Combin. Theory Ser. A, **22**(3):352–360, 1977.
- [2] D. Ellis, *Setwise intersecting families of permutations*, J. Combin. Theory Ser. A, **119**(4):825–849, 2012.
- [3] D. Ellis, E. Friedgut, H. Pilpel, *Intersecting families of permutations*, J. Amer. Math. Soc., **24**(3):649–682, 2011.

A modular equality for m -ovoids of elliptic quadrics

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(Joint work with Klaus Metsch and Francesco Pavese)

An m -ovoid of a finite polar space \mathcal{P} is a set \mathcal{O} of points such that every maximal subspace of \mathcal{P} contains exactly m points of \mathcal{O} . In the case when \mathcal{P} is an elliptic quadric $\mathcal{Q}^-(2r+1, q)$ of rank r in \mathbb{F}_q^{2r+2} , we prove [1] that an m -ovoid exists only if m satisfies

$$F(m) \equiv 0 \pmod{q+1},$$

where

$$F(m) = \begin{cases} m^2 - m & \text{if } r \text{ is odd,} \\ m^2 & \text{if } r \text{ is even and } q \text{ is even,} \\ m^2 + \frac{q+1}{2}m & \text{if } r \text{ is even and } q \text{ is odd,} \end{cases}$$

which rules out many of the possible values of m (previously, only a lower bound on m was known [2]).

References

- [1] A.L. Gavriluk, K. Metsch, F. Pavese, A modular equality for m -ovoids of elliptic quadrics, <https://arxiv.org/abs/2111.07350> arXiv:2111.07350
- [2] J. Bamberg, S. Kelly, M. Law, T. Penttila, Tight sets and m -ovoids of finite polar spaces, *J. Combin. Theory Ser. A*, 114 (2007), no. 7, 1293–1314.

On subspaces of classical polar spaces

Luca Giuzzi

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(Joint work with Ilaria Cardinali and Antonio Pasini)

Let $\Gamma = (\mathcal{P}, \mathcal{L})$ be a non-degenerate embeddable polar space of finite rank $n \geq 2$. Barring two exceptional cases of rank $n = 2$, the space Γ admits the universal embedding $\varepsilon : \Gamma \rightarrow \text{PG}(V)$. A subspace of Γ is a subset \mathcal{S} of the points \mathcal{P} of Γ such that the pointset of any line $\ell \in \mathcal{L}$ of Γ meeting \mathcal{S} in at least two distinct points is fully contained in \mathcal{L} . We say that \mathcal{S} arises from an embedding $\varepsilon : \Gamma \rightarrow \text{PG}(V)$ if there is a subspace X of $\text{PG}(V)$ such that $\mathcal{S} = \varepsilon^{-1}(X)$.

In this talk the following result shall be presented.

Theorem 1 *Let Γ be an embeddable non-degenerate polar space of finite rank $n \geq 2$ admitting universal embedding $\varepsilon : \Gamma \rightarrow \text{PG}(V)$. Then, any subspace \mathcal{S} of Γ which, regarded as polar space, has non-degenerate rank at least 2 arises from ε .*

References

- [1] I. Cardinali, L. Giuzzi and A. Pasini. Nearly all subspaces of a classical polar space arise from its universal embedding, *Lin. Alg. Appl.* **627** (2021), 287-307.

Non-existence of block-transitive subspace designs

Daniel Hawtin

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(Joint work with Jesse Lansdown)

Subspace designs are the q -analogues of balanced incomplete block designs. We prove that there are no nontrivial subspace designs that admit a group of automorphisms acting transitively on the set of blocks of the design [1].

References

- [1] D. R. Hawtin and J. Lansdown. The non-existence of block-transitive subspace designs. *Combinatorial Theory*, 2(1), 2022.

Conditions on Large Caps

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A *cap* in a finite projective space of dimension n over a finite field of order q is a set of points with no three points collinear. It is easy to see that caps have size at most $O(q^{n-1})$, while the largest known constructions for caps have only size $\Theta(q^{\lfloor \frac{2}{3}n \rfloor})$ (as $q \rightarrow \infty$ or $n \rightarrow \infty$). We discuss spectral existence conditions on caps of size $\Omega(q^{\frac{3}{4}n})$.

Cyclic line-spreads and linear spaces

Cian Jameson

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(Joint work with John Sheekey)

There has been much progress towards classifying linear spaces that possess a flag-transitive automorphism group. However, a complete classification is not available, as the case in which the automorphism group is a subgroup of one-dimensional affine transformations remains open; in particular, linear spaces constructed from spreads possessing a transitive automorphism group.

In [1], Pauley and Bamberg constructed new flag-transitive linear spaces via spreads upon which a cyclic group acts transitively, and provided a condition for such spreads to exist in terms of an associated polynomial.

In this talk, we will present our work on describing and classifying the polynomials that give rise to the desired spreads and linear spaces. We will focus on the case of cubic polynomials, corresponding to cyclic line spreads in $\text{PG}(5, q)$, and also discuss some connections with permutation polynomials.

References

- [1] M. Pauley, J. Bamberg. A construction of one-dimensional affine flag-transitive linear spaces, *Finite Fields and their Applications*, (14):537–548, 2008.

The direct sum of q -matroids

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(Joint work with Michela Ceria)

In combinatorics, a q -analogue can be thought of as a generalisation from finite sets to finite dimensional spaces. Sometimes q -analogues are straightforward, but sometimes they are quite counter-intuitive. The reason for this is that the *Boolean lattice*, the poset of a finite set and all its subsets, differs from the *subspace lattice*, the poset of a finite dimensional space and all its subspaces.

In this talk we discuss a not-so-straightforward q -analogue: that of the direct sum of matroids. For classical matroids, the direct sum is one of the most simple methods to make a new matroid out of existing ones. For q -matroids, the q -analogues of matroids, this is a lot less straightforward. The main goal of this talk is to provide an intuitive understanding of this problem, even for those who are not fluent in (q -)matroids.

We will then sketch a definition of the direct sum of q -matroids, using q -polymatroids and the q -analogue of matroid union. We motivate this definition by listing some of its desirable properties.

This talk is based on [1].

References

- [1] Ceria, M. and Jurrius, R. (2021). *The direct sum of q -matroids*. <https://arxiv.org/abs/2109.13637>. Submitted.

On linear codes
associated with the Desarguesian ovoids in $Q^+(7, q)$

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(Joint work with Tao Feng, Peixian Lin and Kai-Uwe Schmidt)

The Desarguesian ovoids in $Q^+(7, q)$, q even, have first been introduced by Kantor by examining the 8-dimensional absolutely irreducible modular representations of $\text{PSL}(2, q^3)$ [1].

We investigate this module for all prime power values of q . The shortest $\text{PSL}(2, q^3)$ -orbit O gives the Desarguesian ovoid in $Q^+(7, q)$ for q even, and it is known to yield a complete partial ovoid of the symplectic polar space $W(7, q)$ for q odd.

In this talk, the hyperplane sections of O will be determined. As a result, the parameters and the weight distribution of the associated \mathbb{F}_q -linear code C_O is derived, and the parameters of the dual code C_O^\perp are computed. After a general discussion of different optimality notions for linear codes, the optimality properties of the codes C_O and C_O^\perp will be investigated. In particular, it will be shown that both codes C_O and C_O^\perp are length-optimal for all prime power values of q .

References

- [1] W. M. Kantor, Ovoids and Translation Planes, *Canad. J. Math.* 34(5) (1982), 1195–1207.

Network Decoding and Packing Problems

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(Joint work with Allison Beemer and Alberto Ravagnani)

Adversarial networks are communication networks where an adversary can act on the network edges, according to some restrictions. This talk is about the 1-shot capacity of such networks, which measures the maximum number of alphabet symbols that can be sent error-free in a single transmission round.

In this talk, I will focus on bounding the size of an error-correcting code in arbitrary adversarial networks, where the final goal is computing the largest possible size of a code that can "defeat" the adversary.

I will describe what the main challenges in this field are and present two bounds on the 1-shot capacity of an adversarial network using a mix of projection and packing arguments. I will then explain how to apply these bounds in concrete examples.

Bivariate semifields and their isotopies

Lukas Kölsch

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(Joint work with Faruk Göloğlu)

Semifields offer one of the classic constructions of translation planes. Many semifields (Dickson semifields, semifields quadratic over a weak nucleus, . . .) use a *bivariate* construction, i.e. they are defined over $\mathbb{F}_{p^{2m}} \cong \mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$ via a semifield multiplication

$$(x, y) * (u, v) = (f(x, y, u, v), g(x, y, u, v)).$$

We focus on specific examples of these bivariate semifields that are constructed via projective polynomials. We present a new family of *commutative* semifields of this form. This is the first family of commutative semifields that contains exponentially (in m) many non-isotopic semifields [1]. We show more generally how it is possible to completely determine when such semifields are isotopic or not using a novel group theoretic approach. This leads to a new improved lower bound for the total number of non-isotopic semifields of odd order [2].

References

- [1] Göloğlu, F., Kölsch, L.: An exponential bound on the number of non-isotopic commutative semifields. <https://arxiv.org/abs/2109.04923>. To appear in *Transactions of the American Mathematical Society*. 2022.
- [2] Göloğlu, F., Kölsch, L.: Counting the number of non-isotopic Taniguchi semifields. <https://arxiv.org/abs/2207.13497>. 2022.

Strong $(t \pmod q)$ arcs in $\text{PG}(k - 1, q)$

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(Joint work with Ivan Landjev and Assia Rousseva)

Several extendability results for linear codes can be explained geometrically using the structure of so-called (strong) $(t \pmod q)$ arcs in $\text{PG}(k - 1, q)$. In this talk we provide a new classification theorem for strong $(3 \pmod 5)$ arcs in $\text{PG}(3, 5)$, where three examples disprove a conjecture of Landjev and Rousseva. The classification is used to show the non-existence of a $[104, 4, 82]_5$ code.

A Note on Sperner's Theorem for Modules over Finite Chain Rings

Ivan Landjev

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(Joint work with Emiliyan Rogachev)

Let \mathcal{P} be a partially ordered set (or poset). We say that the element y of \mathcal{P} covers the element $x \in \mathcal{P}$ if $x \prec y$ and $x \prec y' \prec y$ implies $y = y'$. This is denoted by $x \prec y$. We call \mathcal{P} a ranked poset if there exists a function $r : \mathcal{P} \rightarrow \mathbb{N}_0$ with $r(x) = 0$ for some minimal element $x \in \mathcal{P}$ and $r(y) = r(x) + 1$ for all $x, y \in \mathcal{P}$ with $x \prec y$.

The maximal rank of an element of \mathcal{P} is called the rank of \mathcal{P} . The i -th level of a ranked poset \mathcal{P} is defined by

$$L_i(\mathcal{P}) = \{x \in \mathcal{P} \mid r(x) = i\}.$$

The i -th Whitney number is the cardinality of L_i : $W_i(\mathcal{P}) = |L_i(\mathcal{P})|$. A graded poset is a ranked poset in which all minimal elements have rank 0. A ranked poset is said to have the Sperner property if the maximum cardinality of an antichain is equal to the largest Whitney number.

Let R be a finite chain ring with $|R| = q^m$ and $R/\text{rad } R \cong \mathbb{F}_q$. Consider a finitely generated module ${}_R M$ over R and the partially ordered set $\mathcal{P}(M)$ of all of its (left) submodules. The rank function in ${}_R M$ is defined in the following way: for a submodule ${}_R N < {}_R M$ $r(N) := \log_q |N|$; equivalently, if ${}_R N$ is of shape $m^{\lambda_1}(m-1)^{\lambda_2} \dots 1^{\lambda_m}$ then

$$r(N) = \lambda_1 m + \lambda_2(m-1) + \dots + \lambda_m.$$

We investigate the question of determining the maximal size of an antichain in $\mathcal{P}(M)$.

If M is a free module, i.e. ${}_R M = {}_R R^n$, it turns out that $\mathcal{P}(M)$ has the Sperner property. In particular, the largest antichain contains all submodules N of (poset) rank $r(N) = \frac{mn}{2}$ if at least one of m and n is even, and all submodules of size $r(N) = \lfloor \frac{mn}{2} \rfloor$ or $\lceil \frac{mn}{2} \rceil$ if m and n are odd.

In the case of a non-free module M , the poset $\mathcal{P}(M)$ does not have necessarily the Sperner property. For instance, this is true for

$$M = R \oplus (\theta R)^{n-1}$$

where $\theta \in R \setminus \text{rad } R$, and n is even. In this particular case, a maximal antichain contains submodules of two different poset ranks.

Planes intersecting the Veronese surface in $\text{PG}(5, q)$, q even

Michel Lavrauw

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(Joint work with Nour Alnajjarine)

Let W be the 6-dimensional vector space defined as the set of 2-forms on $\text{PG}(2, q)$. Subspaces of $\text{PG}(W)$ are known as *linear systems of conics* in $\text{PG}(2, q)$. In particular, pencils, nets and webs of conics are the 1-dimensional, 2-dimensional and 3-dimensional subspaces, respectively. Classifying linear systems of conics in $\text{PG}(2, q)$ corresponds to classifying subspaces of $\text{PG}(5, q)$. In 2020, Lavrauw and Sheekey classified lines of $\text{PG}(5, q)$ under the action of the group K of $\text{PGL}(6, q)$ stabilising the Veronese surface, as a consequence they obtained the classification of webs of conics in $\text{PG}(2, q)$ up to projective equivalence. Later, K -orbits of planes intersecting the Veronese surface in $\text{PG}(5, q)$, q odd, were determined by Lavrauw, Popiel and Sheekey, yielding to the classification of rank-1 nets of conics, namely nets with at least one double line. Recently, we considered the equivalent problem of classifying planes meeting the Veronese surface over finite fields of characteristic 2. Particularly, we proved that we have exactly 15 such orbits unless $q = 2$.

In this talk, we elaborate on the connection between subspaces of $\text{PG}(5, q)$ and linear systems of conics. Furthermore, we present a summary of the different types of planes meeting the Veronese surface non-trivially in $\text{PG}(5, q)$, q even. We discuss as well some of their properties and complete invariants. We end by a comparison with the similar classification over finite fields of odd characteristic.

References

- [1] N. Alnajjarine, M. Lavrauw, T. Popiel, “Solids in the space of the Veronese surface in even characteristic.” *Finite Fields and Their Applications*. 83, (2022), 102068.
- [2] N. Alnajjarine and M. Lavrauw, “Planes intersecting the Veronese surface non-trivially in $\text{PG}(5, q)$, q even. In preparation.
- [3] M. Lavrauw and T. Popiel, “The symmetric representation of lines in $\text{PG}(\mathbb{F}_q^3 \otimes \mathbb{F}_q^3)$ ”, *Discrete Math.* **343** (2020) 111775.
- [4] M. Lavrauw, T. Popiel and J. Sheekey, “Nets of conics of rank one in $\text{PG}(2, q)$, q odd”, *J. Geom.* **111** (2020) 36.

Erdős-Ko-Rado results for flags in spherical buildings

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(Joint work with Jan De Beule and Klaus Metsch)

Over the last few years, Erdős-Ko-Rado theorems have been found in many different geometrical contexts including for example sets of subspaces in projective [2] or polar spaces [3]. A recurring theme throughout these theorems is that one can find sharp upper bounds by applying the Delsarte-Hoffman coclique bound to a matrix belonging to the relevant association scheme. In the aforementioned cases, the association schemes turn out to be commutative, greatly simplifying the matter. However, when we do not consider subspaces of a certain dimension but more general flags, we lose this property. In this talk, we will explain how to overcome this problem, using a result originally due to Brouwer [1]. This result, which has seemingly been flying under the radar so far, allows us to derive upper bounds for certain flags in projective spaces and general flags in polar spaces and exceptional geometries. We will show how Chevalley groups, buildings, Iwahori-Hecke algebras and representation theory tie into this story and discuss their connections to the theory of non-commutative association schemes.

References

- [1] Andries Brouwer. The eigenvalues of oppositeness graphs in buildings of spherical type. *Combinatorics And Graphs*. **531** pp. 1-10 (2010).
- [2] Chris Godsil & Karen Meagher. Erdős-Ko-Rado theorems: algebraic approaches. *Cambridge University Press*, Cambridge (2016).
- [3] Valentina Pepe, Leo Storme & Frédéric Vanhove. Theorems of Erdős-Ko-Rado type in polar spaces. *J. Combin. Theory Ser. A*. **118**, 1291-1312 (2011).

Low Boolean degree d functions in Grassmann graphs

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(Joint work with Jan De Beule, Jozefien D'haeseleer and Ferdinand Ihringer)

Consider the n -dimensional vectorspace \mathbb{F}_q^n and consider Boolean functions on the Grassmann graph of k -spaces. One example of such a Boolean function can be constructed as follows: let T be a d -dimensional subspace, then we can define a Boolean function x_T^+ such that $x_T^+(S) = 1$ if and only if $T \subseteq S$.

Secondly, Using these examples, we define a Boolean degree d function on this Grassmann graph as a Boolean function that can be written as a linear combination (over the reals) of all x_T^+ , with $\dim(T) = d$. These Boolean functions appear in a variety of other contexts. One of these connections is the connection with Cameron-Liebler sets of k -spaces. It can be shown that the characteristic function of these sets are precisely Boolean degree 1 functions. More information can be found in [1, 3]. Moreover, this connection resulted in an extended study of Boolean degree 1 functions. Yet for general degree d not so much is known. In this talk we focus on an introduction to these Boolean degree d functions, we look by the connection with Cameron-Liebler sets of k -spaces and end with an exhaustive survey of (non-trivial) examples and non-existence results for Boolean degree 2 functions. This can be found in [2].

References

- [1] A. Blokhuis, M. De Boeck, and J. D'haeseleer, *Cameron-Liebler sets of k -spaces in $PG(n, q)$* , Des. Codes Cryptogr. **87**(8) (2019) 1839–1856.
- [2] J. De Beule, J. D'haeseleer, F. Ihringer and J. Mannaert *Degree 2 Boolean Functions on Grassmann Graphs*, *Elec. J. Combin.*, *submitted*
- [3] Y. Filmus and F. Ihringer, *Boolean degree 1 functions on some classical association schemes*, J. Combin. Theory Ser. A **162** (2019) 241–270.

(Non-)embeddings of the Ree unitals in finite projective planes

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We show that the Ree unital $\mathcal{R}(q)$ has an embedding in a projective plane over a field F if and only if $q = 3$ and \mathbb{F}_8 is a subfield of F . In this case, the embedding is unique up to projective linear transformations. Besides elementary calculations, our proof uses the classification of the maximal subgroups of the simple Ree groups. The main result is the following:

Theorem 1 *Let n be a positive integer, and $q = 3^{2n+1}$. Suppose that Π is a projective plane such that for each embedding $\varphi : \mathcal{R}(3) \rightarrow \Pi$, the image $\varphi(\mathcal{R}(3))$ is contained in a pappian subplane. Then the Ree unital $\mathcal{R}(q)$ has no embedding in Π . In particular, $\mathcal{R}(q)$ has no embedding in a projective plane over a field.*

These results suggest that the problem of projective embeddings of the Ree unitals can be reduced to the question whether the smallest Ree unital has an embedding in a non-desarguesian projective plane. This question is surprisingly hard, even if we assume that the embedding is admissible.

References

- [1] K. Grünig. “Das kleinste Ree-Unital”. In: *Arch. Math. (Basel)* 46.5 (1986), pp. 473–480.
- [2] H. Lüneburg. “Some remarks concerning the Ree groups of type (G_2) ”. In: *J. Algebra* 3 (1966), pp. 256–259.
- [3] A. Montinaro. “On the Ree unital”. In: *Des. Codes Cryptogr.* 46.2 (2008), pp. 199–209.
- [4] G. P. Nagy. “Embeddings of Ree unitals in a projective plane over a field”. In: *Finite Fields and Their Applications* 74 (2021).

Asymptotically good strong blocking sets

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(Joint work with Anurag Bishnoi and Shagnik Das)

Minimal linear codes were first introduced by Cohen and Lempel over the binary field under the name of *linear intersecting codes* [2]. They later gained interest due to their application to secret sharing schemes proposed by Massey [4]. Recently, it has been shown that k -dimensional minimal linear codes in \mathbb{F}_q^n are in one-to-one correspondence with strong blocking sets [1, 5], which are special sets of n points in $\text{PG}(k-1, q)$, such that their intersection with each hyperplane generates the hyperplane itself. The notion of strong blocking set was however already known, since they were originally introduced as a tool for deriving covering codes [3].

In this talk we propose a new general method to construct small strong blocking sets – and hence short minimal linear codes – starting from a set of points in $\text{PG}(k-1, q)$ and a graph with special connectivity properties. In particular, we explore how one can get explicit constructions of families of asymptotically good minimal linear codes, by means of expander graphs and families of asymptotically good linear codes.

References

- [1] G. N. Alfarano, M. Borello, and A. Neri. A geometric characterization of minimal codes and their asymptotic performance. *Adv. in Math. Commun.*, 2020.
- [2] G. Cohen and A. Lempel. Linear intersecting codes. *Discrete Math.*, 56(1):35–43, 1985.
- [3] A. A. Davydov, M. Giulietti, S. Marcugini, and F. Pambianco. Linear nonbinary covering codes and saturating sets in projective spaces. *Adv. in Math. Commun.*, 5(1):119–147, 2011.
- [4] J. L. Massey. Minimal codewords and secret sharing. *In Proceedings of the 6th joint Swedish-Russian international workshop on information theory*, pages 276–279, 1993.
- [5] C. Tang, Y. Qiu, Q. Liao, and Z. Zhou. Full characterization of minimal linear codes as cutting blocking sets. *IEEE Trans. Inform. Theory*, 67(6):3690–3700, 2021.

Small complete caps in $\text{PG}(4n + 1, q)$

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(Joint work with A. Cossidente, B. Csajbók, G. Marino)

Let $\text{PG}(r, q)$ denote the r -dimensional projective space over the finite field with q elements \mathbb{F}_q . A k -cap of $\text{PG}(r, q)$ is a set of k points no three of which are collinear. A k -cap of $\text{PG}(r, q)$ is said to be *complete* if it is not contained in a $(k + 1)$ -cap of $\text{PG}(r, q)$. The study of caps is not only of geometrical interest, but arises from coding theory. Indeed, by identifying the representatives of the points of a complete k -cap of $\text{PG}(r, q)$ with columns of a parity check matrix of a q -ary linear code, it follows that (apart from two sporadic exceptions) complete k -caps of $\text{PG}(r, q)$ with $k > r + 1$ and non-extendable linear $[k, k - r - 1, 4]_q$ 2-codes are equivalent objects.

One of the main issue is to determine the spectrum of the sizes of complete caps in a given projective space and in particular their maximal and minimal possible values. For the size $t_2(r, q)$ of the smallest complete cap in $\text{PG}(r, q)$, the trivial lower bound is $t_2(r, q) > \sqrt{2}q^{\frac{r-1}{2}}$. Apart from the case q even, all known infinite families of complete caps explicitly constructed in $\text{PG}(r, q)$ have size far from the trivial bound. See [1, 2] and references therein.

In this talk I will describe the construction of a complete cap of $\text{PG}(4n + 1, q)$ of size $2(q^{2n} + \dots + 1)$ that is obtained by projecting two disjoint Veronese varieties of $\text{PG}(n(2n + 3), q)$ from a suitable $(2n^2 - n - 2)$ -dimensional projective space, see [3]. This establishes that the trivial lower bound on $t_2(4n + 1, q)$ is essentially sharp.

References

- [1] D. Bartoli, M. Giulietti, G. Marino, O. Polverino, Maximum scattered linear sets and complete caps in Galois spaces, *Combinatorica*, 38 (2018), 255–278.
- [2] M. Giulietti, The geometry of covering codes: small complete caps and saturating sets in Galois spaces, *Surveys in combinatorics* 2013, 51-90. London Math. Soc. Lecture Note Ser., 409, Cambridge Univ. Press, Cambridge, 2013.
- [3] A. Cossidente, B. Csajbók, G. Marino, F. Pavese, Small complete caps in $\text{PG}(4n + 1, q)$, *arXiv:2105.14939*.

On the minimum distance of the $C(\mathbb{H}_{3,6})$ polar Hermitian Grassmann code

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(Joint work with Sarah Gregory, Doel Rivera Laboy and Lani Southern)

We prove that the minimum distance of the polar Hermitian Grassmann code $C(\mathbb{H}_{3,6})$ is $q^9 - q^7$. Our technique is based on partitioning the Polar Hermitian Grassmannian into different sets such that on each partition the code $C(\mathbb{H}_{3,6})$ is identified with evaluations of determinants on sets of Skew-Hermitian matrices. Our bounds come from elementary algebraic methods counting the zeroes of particular classes of polynomials. We extend known results [1] on the minimum distance of polar Hermitian Grassmann codes.

References

- [1] Cardinali, I., Giuzzi, L. Line Hermitian Grassmann codes and their parameters, *Finite Fields and Their Applications* Vol. 51, year 2018, pp. 407-432

The Critical Problem for Combinatorial Geometries and Coding Theory

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The Critical Problem for combinatorial geometries was proposed by Crapo and Rota in the seventies. It asks to compute the largest dimension of a linear subspace that avoid a collection of projective points. This talk will be about the connections between the generalizations of this very classical problem in combinatorial geometry and some fundamental open questions in contemporary coding theory.

Linear Codes, Arcs, Blocking Sets and the Main Problem in Coding Theory

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(Joint work with Ivan Landjev)

Let k and d be positive integers and let $q = p^h$ be a prime power. Write d as $d = sq^{k-1} - a_{k-2}q^{k-2} - \dots - a_1q - a_0$, where $0 \leq a_i < q$ for all i . It is known that the existence of a Griesmer code with parameters $[n, k, d]_q$ is equivalent to that of a blocking multiset (minihyper) in $\text{PG}(k-1, q)$ with parameters $(\sum_{i=0}^{k-2} a_i v_{i+1}, \sum_{i=0}^{k-2} a_i v_i)$ with maximal point multiplicity $s = \lceil d/q^{k-1} \rceil$. Here, as usual, $v_r = \frac{q^r-1}{q-1}$. Minihypers with these parameters always do exist. They can be obtained as the sum of a_{k-2} hyperplanes, a_{k-3} hyperlines, ... , a_0 points. Minihypers produced in this way will be called *canonical*. A second construction starts with a $(\sum_{i=0}^{k-3} a_i v_{i+1}, \sum_{i=0}^{k-3} a_i v_i)$ -minihyper \mathcal{F} in a hyperplane H_0 of $\text{PG}(k-1, q)$. A minihyper \mathcal{F}' is produced in the following way: fix a point P off H_0 and set $\mathcal{F}'(P) = \sum_{i=0}^{k-3} a_i$. For every point $R \neq P$, $\mathcal{F}'(R) = \mathcal{F}(Q)$, where $Q = H_0 \cap \langle P, R \rangle$. The minihyper \mathcal{F}' has parameters $(\sum_{i=0}^{k-2} a_i v_{i+2}, \sum_{i=0}^{k-2} a_i v_{i+1})$. Minihypers produced in this way will be called *lifted*.

We investigate the problem of finding conditions on the numbers a_i that force a minihyper with the above parameters to be canonical or lifted. Interesting special cases of this problem have been considered by Ball, Hill, Landjev, Ward, Storme, Vandendriesche in [1, 2, 3], as well as by Hamada in numerous publications in the early 90's. In this talk, we present steps towards the proof of the following conjecture.

Conjecture. *Let \mathcal{F} be a $(\sum_{i=1}^{k-2} a_i v_{i+1}, \sum_{i=1}^{k-2} a_i v_i)$ -minihyper in $\text{PG}(k-1, q)$, $q = p^h$ prime, where $0 \leq a_i < q$.*

- (i) *If $h = 1$ and $\sum_i a_i \leq q - 1$, then \mathcal{F} is canonical or lifted;*
- (ii) *if $h > 1$ and $\sum_i a_i \leq q - q/p$, then \mathcal{F} is canonical or lifted.*

References

- [1] S. Ball, R. Hill, I. Landjev, H. N. Ward, On $(q^2 + q + 2, q + 2)$ -arcs in the projective plane $\text{PG}(2, q)$, Des. Codes Cryptogr. 24(2001), 205–224.
- [2] I. Landjev, L. Storme, A study of $(x(q+1), x; 2, q)$ -minihypers, Des. Codes Cryptogr. 54(2010), 135–147.
- [3] I. Landjev, P. Vandendriesche, A study of (xv_t, xv_{t-1}) -minihypers in $\text{PG}(2, t)$, J. Combin. Theory Ser A 119(2012), 1123–1131.

Minimum size linear sets

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(Joint work with Vito Napolitano, Olga Polverino and Ferdinando Zullo)

Linear sets are natural generalization of subgeometries and, in recent years, they have been intensively used to construct and classify several geometrical and algebraic objects. In [1], De Beule and Van de Voorde proved a lower bound on the size of a linear set. In this talk we will deal with linear sets attaining this bound in the projective line. Examples of these linear sets have been found by Lunardon and Polverino in 2000 and, more recently, by Jena and Van de Voorde in [2]. Classification results for linear sets having minimum size of rank k are known only for $k \leq 5$. First, we will provide classification results for linear sets of minimum size when n is prime. When n is not a prime, we will show how to construct minimum size linear sets that cannot be obtained as instances of the construction given in [2]. The talk is based on [3].

References

- [1] J. DE BEULE, G. VAN DE VOORDE: The minimum size of a linear set, *Journal of Combinatorial Theory, Series A*, **164**:109–124, 2019.
- [2] D. JENA, G. VAN DE VOORDE: On linear sets of minimum size, *Discrete Mathematics*, **344**(3):112230, 2021.
- [3] V. NAPOLITANO, O. POLVERINO, P. SANTONASTASO AND F. ZULLO: Classifications and constructions of minimum size linear sets, *arXiv:2201.02003*.

Designs in finite general linear groups

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(Joint work with Alena Ernst)

It is known that the notion of a transitive subgroup of a permutation group G extends naturally to subsets of G . This talk is about subsets of the general linear group $\mathrm{GL}(n, q)$ acting transitively on flag-like structures, which are common generalisations of t -dimensional subspaces of \mathbb{F}_q^n and bases of t -dimensional subspaces of \mathbb{F}_q^n . I shall discuss structural characterisations of transitive subsets of $\mathrm{GL}(n, q)$ using the character theory of $\mathrm{GL}(n, q)$ and interpret such subsets as designs in the conjugacy class association scheme of $\mathrm{GL}(n, q)$. While transitive subgroups of $\mathrm{GL}(n, q)$ are quite rare, it will be shown that, for all fixed t , there exist nontrivial subsets of $\mathrm{GL}(n, q)$ that are transitive on linearly independent t -tuples of \mathbb{F}_q^n , which also shows the existence of nontrivial subsets of $\mathrm{GL}(n, q)$ that are transitive on more general flag-like structures. These results can be interpreted as q -analogs of corresponding results for the symmetric group.

Partial permutation decoding of the binary code of the projective plane $\text{PG}(2, q)$, q even

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(Joint work with D. Crnković, N. Mostarac and B.G. Rodrigues)

(Partial) permutation decoding was introduced by F.J. MacWilliams [2]. The algorithm uses sets of code automorphisms, called *PD-sets*, that are defined with respect to a given information set of the code. The idea of the algorithm is to move the error positions outside of the information set positions, to correct the errors.

The goal is to construct PD-sets as small as possible. An *s-PD-set* is a set of code automorphisms which can correct s errors that occurred during the transmission of a codeword.

In [3], P. Vandendriessche constructed a particular basis for the binary code arising from the incidence matrix of the projective planes $\text{PG}(2, q)$, $q = 2^h$, $5 \leq h \leq 9$.

Using this basis, we construct a 2-PD-set of size 16 and a 3-PD-set of size 75 for the binary code arising from the incidence matrix of the projective planes $\text{PG}(2, q)$, $q = 2^h$, $5 \leq h \leq 9$ [1].

References

- [1] D. Crnković, N. Mostarac, B.G. Rodrigues, and L. Storme, *s*-PD-sets for codes from projective planes $\text{PG}(2, 2^h)$, $5 \leq h \leq 9$. *Adv. Math. Commun.* **15** (2021), 423-440.
- [2] F.J. MacWilliams, Permutation decoding of systematic codes. *Bell Syst. Tech. J.* **43** (1964), 485-505.
- [3] P. Vandendriessche, Codes of Desarguesian projective planes of even order, projective triads and $(q + t, t)$ -arcs of type $(0, 2, t)$. *Finite Fields Appl.* **17** (2011), 521-531.

Renitent lines

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(Joint work with B. Csajbók and Zs. Weiner)

One of the key motivations in the history of finite geometries is the study of symmetric structures, i.e. structures admitting a large symmetry group. These structures (quadrics, Hermitian varieties, subgeometries over a subfield, etc.) are typically very “regular” when you consider their intersection properties with the subspaces of the ambient geometry; and there exist many “classification-type” results, stating that an “intersection-wise very regular” set must be one on the list of the (classical, symmetric) structures.

A natural next step is to investigate point sets, which behave “almost regularly” with respect to the subspaces of the ambient space. In this talk we restrict ourselves to point sets of a desarguesian affine plane $AG(2, q)$, where q is a power of the prime p (although we have natural but not obvious extensions to other spaces.) It may well happen, that our point set intersects almost all lines of a parallel class in the same number of points (possibly mod p). If it happens for many parallel classes then one may guess that the reason is that our point set has a hidden structure, i.e. the non-regular intersections may be “corrected”, or at least they also possess some regularity themselves.

Now we define **renitent lines**.

Definition. Let \mathcal{M} be a multiset of $AG(2, q)$. For some integer $\lambda \leq (q - 1)/2$ a direction (d) is called $(q - \lambda)$ -uniform if there are at least $(q - \lambda)$ affine lines with slope d meeting \mathcal{M} in the same number of points modulo p . This number will be called the *typical intersection number* at (d) . The rest of the lines with direction (d) will be called *renitent*.

Note that different directions may have different typical intersection numbers.

Some general versions of the following theorem will be presented in the talk, roughly saying that under natural assumptions, the renitent lines are contained in a curve of the dual plane, of relatively small degree:

Theorem. Take a multiset \mathcal{T} of $AG(2, q)$ and let \mathcal{E}_λ denote a set of at most q directions which are $(q - \lambda)$ -uniform such that the following hold:

- (i) $0 < \lambda \leq \min\{q - 2, p - 1\}$,
- (ii) for each $(d) \in \mathcal{E}_\lambda$ the renitent lines meet \mathcal{T} in the same number, say t_d , of points modulo p ,
- (iii) for each $(d) \in \mathcal{E}_\lambda$ if m_d denote the typical intersection number at direction (d) , then $t_d - m_d \pmod p$ does not depend on the choice of (d) .

Then the renitent lines with direction in \mathcal{E}_λ are contained in an algebraic envelope of class λ .

Generalized Korchmáros-Mazzocca arcs and renitent lines

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(Joint work with Bence Csajbók and Péter Sziklai)

Korchmáros-Mazzocca arcs are point sets of size $q + t$ intersecting each line in 0, 2 or t points in a finite projective plane of order q . When $t \neq 2$, this means that each point of the point set is incident with exactly one line meeting the point set in t points. For $t = 1$, we get the ovals, for $t = 2$ the hyperovals; thus this concept generalizes well-known objects of finite geometry. They were introduced and first studied by Korchmáros and Mazzocca in 1990, see [3]. In [1], with Bence Csajbók, we generalized the concept of Korchmáros-Mazzocca arcs, namely in $\text{PG}(2, q)$, we changed 2 in the definition above to any integer m . Also, we introduced the mod p variants of these objects. In this talk, I will give examples and some characterization type result on these objects, for example I will describe all examples when m or t is not divisible by p . Under some condition, we also proved the existence of a nucleus. In order to do so, we had to show that 'the renitent' lines (the t -secants) through the points of an m -secant have a nucleus (and a similar lemma hold for the mod p variant of the problem). Recently, together with Bence Csajbók and Péter Sziklai ([2]), we studied possible generalization of the above phenomenon, i.e. we investigated point sets of a desarguesian affine plane, for which there exist some (sometimes: many) parallel classes of lines, such that almost all lines of one parallel class intersect our set in the same number of points (possibly mod p , the characteristic). We proved results on the (regular) behaviour of the lines with exceptional intersection numbers. In this talk, I will also give some insight into this study.

References

- [1] B. Csajbók, Zs. Weiner, Generalizing Korchmáros-Mazzocca arcs, *Combinatorica*, **41** (2021) 601–623.
- [2] B. Csajbók, P. Sziklai, Zs. Weiner, Renitent lines, submitted.
- [3] G. Korchmáros and F. Mazzocca, On $(q + t)$ -arcs of type $(0, 2, t)$ in a Desarguesian plane of order q . *Math. Proc. Cambridge Philos. Soc.* **108** (3) (1990), 445–459.

The linear programming bounds in classical association schemes

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(Joint work with Kai-Uwe Schmidt)

Many interesting codes such as q -ary codes, rank-metric codes, and subspace codes can be viewed as subsets of association schemes. The corresponding association schemes are the Hamming scheme, the Johnson scheme, and several q -analogs of them. Based on the theory of association schemes, Delsarte introduced a linear program that yields an upper bound for the size of codes. This linear program has been studied for many years in the case of the Hamming scheme and the Johnson scheme, but it is still unknown what the optimal solutions of their linear programs look like. By using a unified way, we will give the optimal solution of the linear program for codes in the projective space, in the bipartite halves (Greeks and Latins) of the hyperbolic polar space, and in one of the Hermitian polar spaces, as well as for their affine counterparts: bilinear forms scheme, alternating forms scheme, and Hermitian forms scheme.

Moreover, the bounds for the bipartite halves and for one of the Hermitian polar spaces can be used to derive bounds for codes in all the remaining polar spaces. These bounds will be used to give an almost complete classification of t -Steiner systems in polar spaces.

A standard form for scattered linearized polynomials and properties of the related translation planes

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(Joint work with Giovanni Longobardi)

A scattered polynomial is an \mathbb{F}_q -linearized polynomial $f(x) = \sum_{i=0}^{n-1} a_i x^{q^i} \in \mathbb{F}_{q^n}[x]$ such that for any $y, z \in \mathbb{F}_{q^n}$ the condition $zf(y) - yf(z) = 0$ implies that y and z are \mathbb{F}_q -linearly dependent. When $f(x)$ is a scattered polynomial, then $U_f = \{(x, f(x)) : x \in \mathbb{F}_{q^n}\}$ is a scattered \mathbb{F}_q -subspace with respect of the Desarguesian spread $\mathcal{D} = \{\langle v \rangle_{\mathbb{F}_{q^n}} : v \in \mathbb{F}_{q^n}^2 \setminus \{(0, 0)\}\}$ of $\mathbb{F}_{q^n}^2$; that is, $\dim_{\mathbb{F}_q} (\langle v \rangle_{\mathbb{F}_{q^n}} \cap U_f) \leq 1$ for any $v \in \mathbb{F}_{q^n}^2$. The collections of \mathbb{F}_q -subspaces $\mathcal{H}_f = \{\langle (y, f(y)) \rangle_{\mathbb{F}_{q^n}} : y \in \mathbb{F}_{q^n} \setminus \{0\}\}$ and $\mathcal{H}'_f = \{hU_f : h \in \mathbb{F}_{q^n} \setminus \{0\}\}$ cover the same vector set. As a consequence, $(\mathcal{D} \setminus \mathcal{H}_f) \cup \mathcal{H}'_f$ is a spread of the \mathbb{F}_q -vector space $\mathbb{F}_{q^n}^2$, and gives rise to a translation plane \mathcal{A}_f , whose general properties have been described in [1].

In this talk I will present results concerning the stabilizer G_f of the subspace U_f , $f(x)$ a scattered linearized polynomial in $\mathbb{F}_{q^n}[x]$, under the action of $\text{GL}(2, q^n)$. Each G_f contains at least the $q - 1$ maps $(x, y) \mapsto (ax, ay)$, $a \in \mathbb{F}_q \setminus \{0\}$. The elements in G_f are simultaneously diagonalizable. This has several consequences: (i) the polynomials such that $|G_f| > q - 1$ have a standard form of type $\sum_{j=0}^{n/t-1} a_j x^{q^{s+jt}}$ for some s and t such that $(s, t) = 1$, $t > 1$ a divisor of n ; (ii) this standard form is essentially unique; (iii) the translation plane \mathcal{A}_f associated with $f(x)$ admits affine homologies if and only if $|G_f| > q - 1$, and in that case the affine homologies with axis through the origin form two groups of cardinality $(q^t - 1)/(q - 1)$ that exchange axes and coaxes; (iv) no plane of type \mathcal{A}_f , $f(x)$ a scattered polynomial not of pseudoregulus type, is a generalized André plane.

References

- [1] V. Casarino, G. Longobardi, C. Zanella, Scattered linear sets in a finite projective line and translation planes, *Linear Algebra Appl.* 650 (2022), 286–298.

Subcovers of generalized GK curves and their automorphism groups

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(Joint work with Maria Montanucci, Guilherme Tizziotti)

The \mathbb{F}_{q^6} -maximal GK curve was constructed in [3] as the first example of maximal curve which is not covered by the corresponding Hermitian curve. Two $\mathbb{F}_{q^{2n}}$ -maximal generalizations of the GK curve were provided for any odd n , the first one by Garcia, Güneri and Stichtenoth [2], and the second one by Beelen and Montanucci [1]. Subcovers \mathcal{X}_1 of the first generalized GK curve were defined in [4] as examples of new maximal curves which are not covered by the Hermitian curve.

We construct analogous subcovers \mathcal{X}_2 of the second generalized GK curve, and obtain new maximal curves not covered by the Hermitian curve. Also, we determine the full automorphism groups of the curves \mathcal{X}_1 and \mathcal{X}_2 , which are related to the automorphism group $\text{PGU}(3, q)$ of the \mathbb{F}_{q^2} -maximal Hermitian curve. This provides a new characterization of the GK curve in terms of its automorphism group.

References

- [1] P. Beelen and M. Montanucci, *A new family of maximal curves*, J. Lond. Math. Soc. (2) 98 (2018), no. 3, 1–20.
- [2] A. Garcia, C. Güneri, and H. Stichtenoth, *A generalization of the Giulietti-Korchmáros maximal curve*, Adv. Geom. 10 (2010). no. 3, 427–434.
- [3] M. Giulietti and G. Korchmáros, *A new family of maximal curves over a finite field*, Math. Ann. 343 (2009), 229–245.
- [4] S. Tafazolian, A. Teherán-Herrera, and F. Torres, *Further examples of maximal curves which cannot be covered by the Hermitian curve*, J. Pure Appl. Algebra, 220 (2016), no. 3, 1122–1132.

Identifiable Waring subspaces over finite fields

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(Joint work with Michel Lavrauw)

Waring’s problem, of expressing an integer as the sum of powers, has a very long history going back to the 17th century [6], and the problem has been studied in many different contexts. A special case of this is one of the classical problems for symmetric tensors (multilinear forms): determine the minimum integer k such that a generic symmetric tensor $f \in \text{Sym}^d(V)$ can be written as the sum of k pure tensors of $\text{Sym}^d(V)$. This problem is a reformulation of writing a homogeneous polynomial f of degree d as the sum of d -th powers of linear forms, and hence it can be seen as a generalisation of the problem posed by Waring. The connection is given by the correspondence between homogeneous polynomials of degree d in $\mathbb{F}[X_0, \dots, X_n]$ and the elements of $\text{Sym}^d(V)$. The value k is called the *Waring rank of f* and the decomposition of f into the sum of k pure tensors is called a *Waring decomposition*. If the linear forms appearing in a minimal decomposition are unique, up to a nonzero scalar multiple, then f is called *Waring identifiable*. The question of identifiability is naturally interesting on its own and has many applications. When \mathbb{F} is the field of complex numbers, the Waring rank of a *generic* form in $\text{Sym}^d(\mathbb{C}^{n+1})$ was determined by Alexander and Hirschowitz in [1], see [3, 5, 2] for more recent results.

In this talk we introduce the notion of a *Waring subspace* and a *Waring identifiable subspace* with respect to a projective algebraic variety \mathcal{X} . When \mathcal{X} is the Veronese variety, these subspaces play a fundamental role in the theory of symmetric tensors and are related to the Waring decomposition and Waring identifiability problem of symmetric tensors (homogeneous polynomials).

References

- [1] J. ALEXANDER AND A. HIRSCHOWITZ: Polynomial interpolation in several variables. *J. Algebraic Geom.* **4** (1995), no. 2, 201–222.
- [2] E. BALLICO: On the weak non-defectivity of Veronese embeddings of projective spaces, *Cent. Eur. J. Math.* **3** (2005), no. 2, 183–187.
- [3] L. CHIANTINI, G. OTTAVIANI, N. VANNIEWENHOVEN: On generic identifiability of symmetric tensors of subgeneric rank. *Trans. Amer. Math. Soc.* **369** (2017), no. 6, 4021–4042.
- [4] M. LAVRAUW AND F. ZULLO: Waring subspaces of algebraic varieties over finite fields, arXiv:2207.13456.
- [5] K. RANESTAD AND C. VOISIN: Variety of power sums and divisors in the moduli space of cubic fourfolds, *Doc. Math.* **22** (2017), 455–504.
- [6] E. WARING: *Meditationes Algebraicae*, Cambridge: J. Archdeacon. (1770).

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