The André/Bruck-Bose representation of a linear set on a projective line

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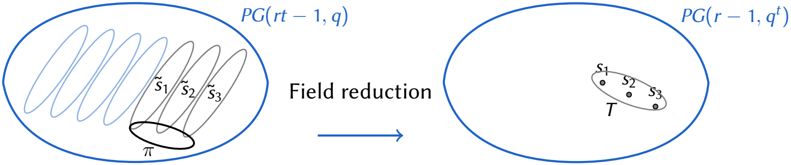
2 Outline

1. Introduction
2. The ABB representation of a linear set on a line
3. Corollary for higgledy-piggledy sets

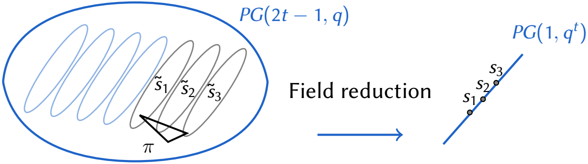
4 Linear sets

Definition

An $\mathbb{F}_q$-linear set of rank k is a set T of points of $\mathrm{PG}(r-1, q^t)$ s.t. there exists a $(k-1)$-space π in $\mathrm{PG}(rt-1, q)$ such that the points of T correspond to the elements of $\mathcal{X}$ that have a non-empty intersection with π.



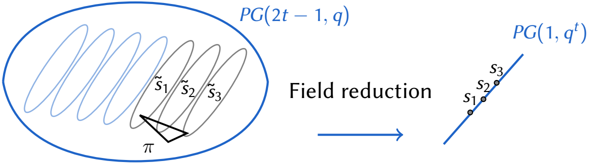
Linear sets of rank 3 on a line



Two types

- Scattered linear sets,
- Clubs with head point H.

5 Linear sets of rank 3 on a line



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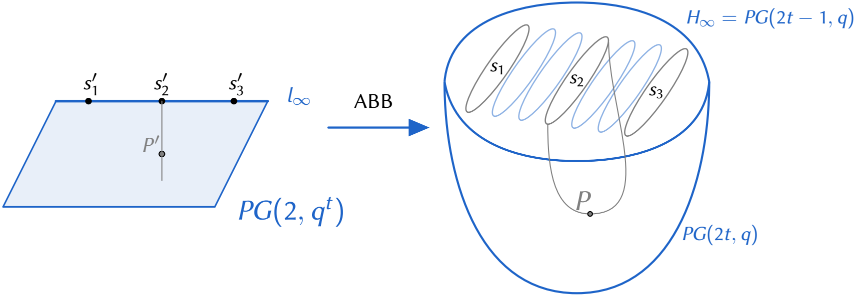
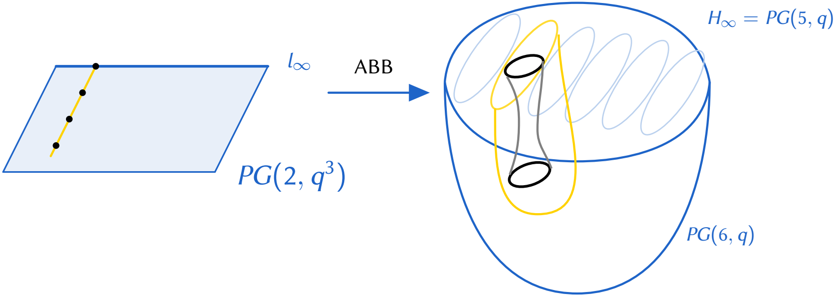
6 André/Bruck-Bose construction



$s_1, s_2, s_3 \in S$,
S is $(t-1)$-spread in H_∞

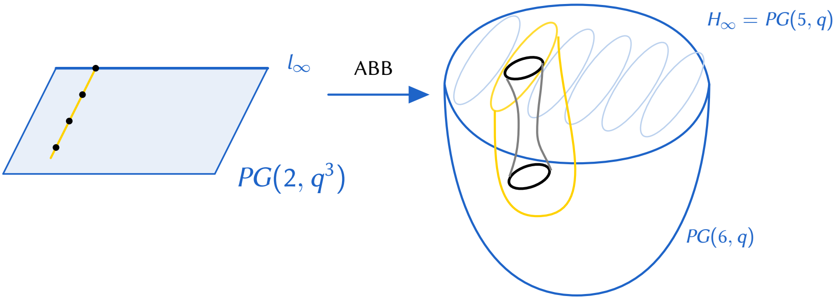
8 Tangent scattered linear set in $\mathrm{PG}(1, q^3)$



Theorem

Let S be the affine point set of a tangent scattered linear set of rank 3 in $\mathrm{PG}(1, q^3)$, then the ABB-representation of S is the affine part of a hyperbolic quadric $\mathcal{Q}$ intersecting the plane π_∞ in a conic.

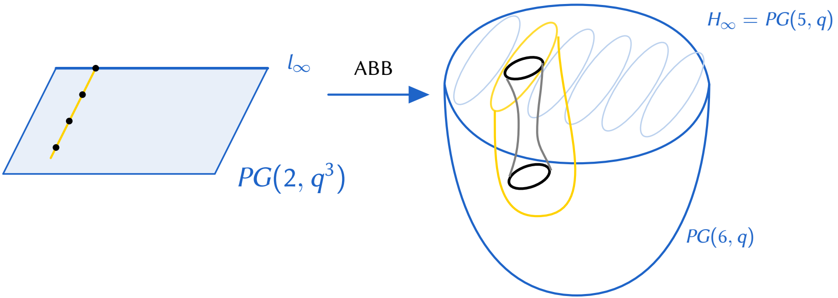
Tangent scattered linear set in $\mathrm{PG}(1, q^3)$



We know even more

- *Special* hyperbolic quadric
- Special hyperbolic quadric always comes from tangent scattered linear set.

Tangent scattered linear set in $\mathrm{PG}(1, q^3)$



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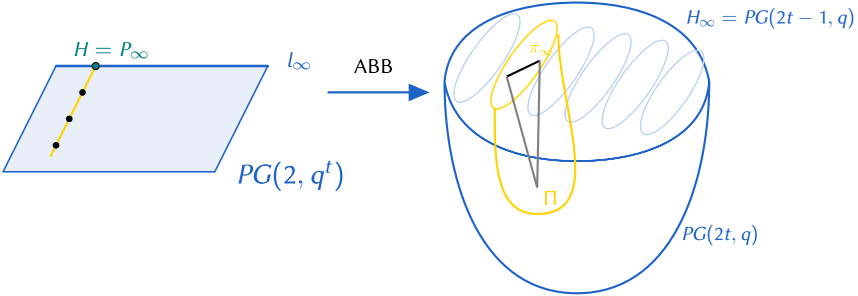
10 Tangent club

- Club with head point $H \in l_\infty$.
- Club with head point $H \notin l_\infty$.

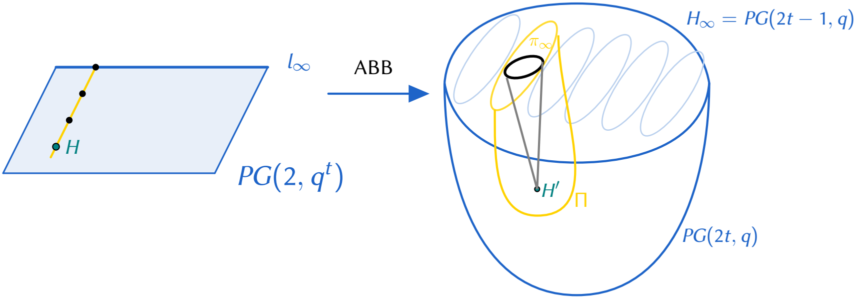
11 Tangent club with $H \in l_\infty$



Theorem

Let S be the affine point set of a tangent club of rank 3 in $\mathrm{PG}(1, q^t)$ with $H = P_\infty$, then the ABB-representation of S is an affine plane in Π.

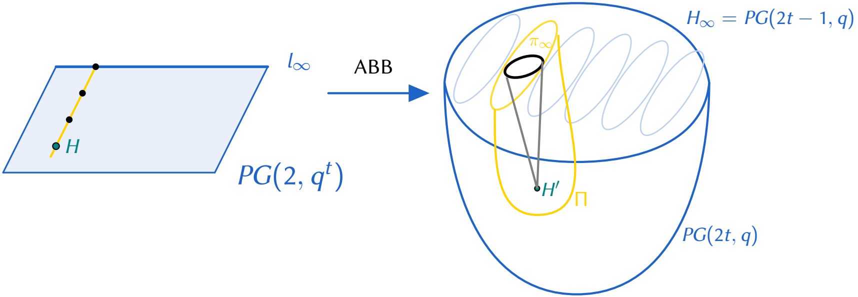
12 Tangent club with $H \notin l_\infty$



Theorem

Let S be the affine point set of a tangent club of rank 3 in $\mathrm{PG}(1, q^t)$, t prime and $P_\infty \neq H$, then the ABB-representation of S is the affine part of a cone in Π with vertex H' and base a NRC of degree $t-1$ contained in π_∞.

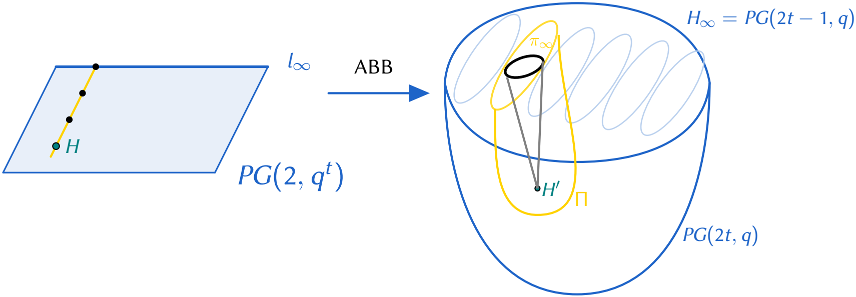
Tangent club with $H \notin l_\infty$



We know even more

- *Special* cone.
- Special cone always comes from tangent club of rank 3 with $H \neq P_\infty$.
- Generalisation for rank $k \geq 3$.

Tangent club with $H \notin l_\infty$



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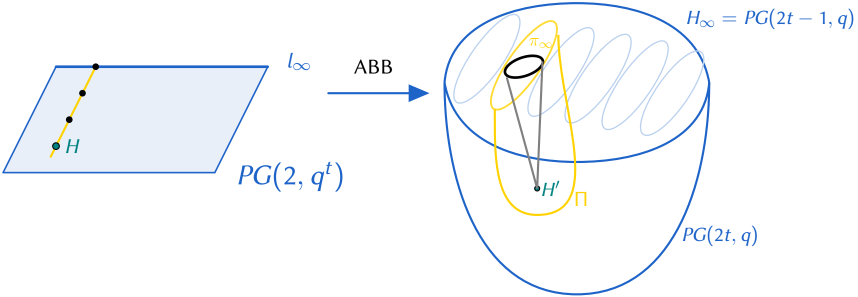

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14 The proof

Techniques

- Projection arguments
- Coordinates
- Combinatorial arguments

Proof for tangent club with $H \notin l_\infty$, t prime

Result [S. Rottey, J. Sheekey, G. Van de Voorde, 2015]

ABB representation of $\mathbb{F}_q$-sublines.

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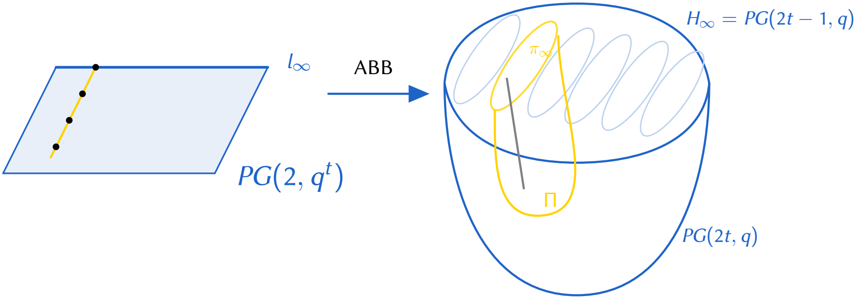
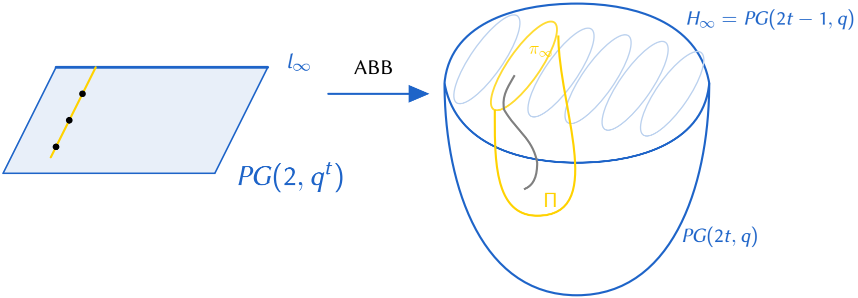


Tangent $\mathbb{F}_q$-subline $\rightarrow$ Line in Π.

External $\mathbb{F}_q$-subline $\rightarrow$ Normal rational curve in Π.

ABB representation of $\mathbb{F}_q$-sublines



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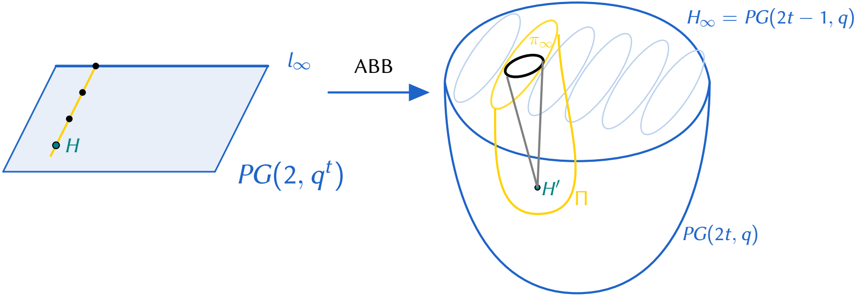
Proof for tangent club with $H \notin l_\infty$, t prime

Result [M. Lavrauw, G. Van de Voorde, 2010]

Intersection of sublines and linear sets on a line

The proof



Result

Through 2 non-head points of a club S of $\mathrm{PG}(1, q^t)$, there is exactly one subline contained in S, which contains the head of the club.

Higgledy-piggledy planes in $\mathrm{PG}(5, q)$

Definition

A set of planes is a hig-pig set of planes in $\mathrm{PG}(5, q)$ if the set of intersection points of these planes with any solid μ in $\mathrm{PG}(5, q)$ spans μ itself.

Under field reduction

A set of points in $\mathrm{PG}(1, q^3)$, not contained in a linear set of rank max 3.

Known lower bound by L. Denaux: 7 planes.

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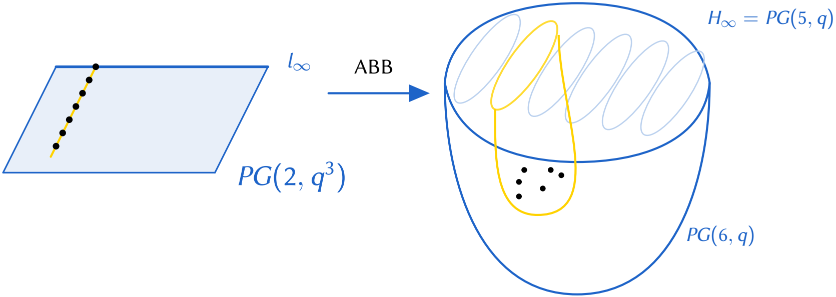
21 Higgledy-piggledy planes in $\mathrm{PG}(5, q)$



Thank you very much for your attention.