Aart and Combinatorics

Celebrating the work of Aart Blokhuis

20–24 February 2023

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Abstracts

Bent and hyperbent functions from hyperovals

Kanat Abdukhalikov

UAE University

(Joint work with Duy Ho)

We consider descriptions of hyperovals in terms of g-functions (using polar coordinates), and employ them to construct bent and hyperbent functions. In order to characterize hyperbentness of the obtained functions, Kloosterman sums are used. From a geometric viewpoint, our construction is a generalization of Dillon's bent functions.

Edge domination in incidence graphs

Sam Adriaensen

Vrije Universiteit Brussel

(Joint work with Sam Mattheus and Sam Spiro)

The *edge domination number* $\gamma_e(G)$ of a graph G is the size of the smallest subset S of its edges, such that any edge in G intersects some edge of S. In this talk, we will discuss the edge domination number of incidence graphs of some nice incidence structures. In particular, the following result is central.

Theorem 0.1. ([1]) *Let G be the incidence graph of a symmetric $2-(v,k,\lambda)$ design D. Then $v - \gamma_e(G)$ equals the largest number α such that D contains a set X of α points and a set Y of α blocks, with no point of X incident with a block of Y.*

This leads us to explore upper and lower bounds on α.

[1] S. Spiro, S. Adriaensen, S. Mattheus. Incidence-free sets and edge domination in incidence graphs. arXiv:2211.14339 (2022).

The linear blocking set conjecture

Simeon Ball

Universitat Politécnica Catalunya

In this talk I will review Aart's proof of the conjecture that all small blocking sets in Desarguesian projective planes of prime order are trivial, i.e. they contain a line, from [AB]. I will go on to talk about possible approaches to proving the conjecture that all small blocking sets in Desarguesian projective planes are linear and review some of Aart's other results.

[AB] Blokhuis, Aart, On the size of a blocking set in PG$(2, p)$. *Combinatorica*, **14** (1994), no. 1, 111–114.

Affine blocking sets and trifferent codes

Anurag Bishnoi

Delft University of Technology

(Joint work with Jozefien D'haeseleer, Dion Gijswijt and Aditya Potukuchi)

We prove new upper and lower bounds on the smallest size of a set of points in a finite affine space that meets every affine subspace of a fixed codimension. Our lower bounds are based on bounds in coding theory and our upper bounds combine a geometrical argument with the probabilistic method. We prove an equivalence between symmetric affine blocking sets for subspaces of codimension 2, over the finite field $\mathbb{F}_3$, and linear perfect 3-hash codes, which are also known as trifferent codes. Using this equivalence, we prove that any linear trifferent code of length n has size at most $3^{n/4.25}$ and show the existence of such codes with size $(9/5)^{n/4}$, thus matching the best known lower bound on non-linear trifferent codes.

We also give explicit constructions of small affine blocking sets with respect to codimension-2 subspaces in $\mathbb{F}_q^n$, for every fixed q and n large enough. Our construction relies on expander graphs and asymptotically good q-ary codes. By restricting to $q = 3$, we obtain new explicit constructions of trifferent codes.

Things I wanted to prove but couldn't

Aart Blokhuis

There are a couple of problems in finite geometry that were always close to my heart. I know the answer, I know it is provable, but I never managed to find the final lines. I encourage the audience to work on it, and make me happy by solving one or more of them.

Back to the Future

Robert Calderbank

Duke University

In 1948, Shannon created the field of Information Theory when he published A Mathematical Theory of Communication in the Bell System Technical Journal. He established channel capacity as the fundamental limit on the efficiency of communication over noisy channels, and presented the challenge of finding specific families of codes that achieve that limit.

Shannon starts with a channel and asks how to maximize mutual information between inputs and outputs, statisticians start with experiments that reveal the channel connecting inputs and outputs, and mathematicians start with the question of how many spheres can touch a given sphere in N dimensions. I will describe how all paths lead to Reed-Muller codes.

As computation became more possible, coding theory changed character, symmetry faded from consciousness, and the focus shifted to understanding the dynamics of iterative decoding algorithms. I will argue that geometry still matters by sketching how symmetries of Reed Muller codes lead to a proof that they achieve capacity on erasure channels under bit map decoding.

Today the nature of computation is changing as quantum devices move out of physics labs and become generally programmable. I will describe how classical Reed Muller codes can be used to enable resilient quantum computation.

Intersection problems in projective geometries

Jozefien D'haeseleer

Ghent University

In the last decades, projective subspaces, pairwise intersecting in at least a t-space were investigated. The case with $t = 0$ (the so called Erdős-Ko-Rado-sets), received special attention [DBS], [GM]. Let PG(n, q) be the projective space of dimension n over the finite field of order q. Frankl and Wilson proved that the largest set of k-spaces, pairwise intersecting in at least a t-space in $PG(n, q)$, $0 \leq t \leq k, n \geq 2k + 2$, is the family of all k-spaces containing a fixed t-space [FW]. This example is also called a *t-pencil*.

In the first part of this talk, I will discuss the structure of maximal families of k-spaces of PG(n, q), pairwise intersecting in at least a t-space, that are different from the pencil example. The classification of the second largest maximal families will be given for general values of t [D]. This problem is also called the Hilton-Milner problem, refering to the authors that solved the same, orginինal question in the context of set theory [HM]. For $t = k - 2$ we give a more detailed classification describing the largest 9 families of k-spaces, pairwise intersecting in at least a $(k - 2)$-space [DLRS].

The topic, discussed in the second part of the talk, can be described within the context of subspace codes. While in the first part we look at families of subspaces that pairwise intersect in *at least* a t-space, in this part we look at families of k-spaces that pairwise intersect in *exactly* a t-space. We also refer to these families as *t-intersecting constant dimension codes*, or abbreviated *SCID's*. The classical example of a t-intersecting constant dimension code is the set of k-spaces pairwise intersecting in a fixed t-space α, which is called a *Sunflower* through α. Within the theory on t-intersecting constant dimension codes, it is a known result that large t-intersecting constant dimension codes are equal to Sunflowers. More precisely, the following result is known.

Theorem 0.2. [ER] *Let C be a t-intersecting constant dimension code of k-dimensional spaces in PG(n, q), where*

$$|C| > \left(\frac{q^{k+1} - q^{t+1}}{q - 1}\right)^2 + \left(\frac{q^{k+1} - q^{t+1}}{q - 1}\right) + 1,$$

then C is a sunflower.

This lower bound is called the *Sunflower bound* and it is generally believed that the lower bound of the preceding theorem is too large. This motivates the research to improve this lower bound. In this second part of my talk, I will present an improvement on the Sunflower bound for k-spaces, pairwise intersecting in a projective point.

[DBS] M. De Boeck and L. Storme, Theorems of Erdős-Ko-Rado type in geometrical settings. Science China Math. 56, 1333–1348, (2013).

[D] J. D'haeseleer, Hilton-Milner results in projective and affine spaces. ArXiv:2007.15851 (Submitted 2021).

[DLRS] J. D'haeseleer, G. Longobardi, A. Riet, and L. Storme, Maximal sets of k-spaces pairwise intersecting in at least a $(k-2)$-space. ArXiv:2005.05494 (Submitted 2021).

[ER] T. Etzion and N. Raviv, Equidistant codes in the Grassmannian. Discrete Appl. Math. 186, 87–97, (2015).

[FW] P. Frankl and R. M. Wilson, The Erdős-Ko-Rado theorem for vector spaces. J. Combin. Theory Ser. A 43, 228–236, (1986).

[GM] C. Godsil and K. Meagher, Erdős-Ko-Rado Theorems: Algebraic Approaches. Cambridge University Press (2015).

[HM] A. J. W. Hilton and E. C. Milner, Some intersection theorems for systems of finite sets. Quart. J. Math. Oxford Ser.(2), 18, 369–384, (1967).

On strongly regular graphs decomposable into a divisible design graph and a coclique

Alexander Gavrilyuk

Shimane University

(Joint work with Vladislav Kabanov)

We will discuss a generalization of the construction of strongly regular graphs, presented in ([1]). It starts with a divisible design graph (which can be obtained from a variation of the Wallis – Fon-Der-Flaass prolific construction ([2])) and extends it to a strongly regular graph by adding a coclique whose size is to satisfy the Hoffman-Delsarte bound.

[1] V.V. Kabanov, A new construction of strongly regular graphs with parameters of the complement symplectic graph, arXiv:2203.03921v2.

[2] V.V. Kabanov, New versions of the Wallis – Fon-Der-Flaass construction to create divisible design graphs, Discrete Math., 345:113054, 2022.

The Seidel matrix

Willem Haemers

In 1966 Jaap Seidel introduced the (-1,1,0) adjacency matrix of a graph, now called the Seidel matrix. The spectrum of a Seidel matrix is invariant under Seidel switching, a graph operation which defines an equivalence relation. Equivalence classes correspond to sets of equiangular lines and so-called two-graphs and the spectrum of the Seidel matrix plays an important role in the study of these objects. In later years the Seidel matrix and its spectrum has received attentions for other reasons which will be discussed in the talk. We pay attention to the Seidel energy, characterisations by the spectrum, and the connection with signed graphs.

The Eckardt point configuration of cubic surfaces revisited

Fatma Karaoglu

Gebze Technical University

(Joint work with Anton Betten)

The classification problem for cubic surfaces with 27 lines is concerned with describing a complete set of the projective equivalence classes of such surfaces. Despite a long history of work, the problem is still open. One approach is to use a coarser equivalence relation based on geometric invariants. The Eckardt point configuration is one such invariant. It can be used as a coarse-grain case distinction in the classification problem. We provide an explicit parametrization of the equations of cubic surfaces with a given Eckardt point configuration over any field. Our hope is that this will be a step towards the bigger goal of classifying all cubic surfaces with 27 lines.

Knots and Codes

Eindhoven University of Technology

Altan Berdan Kılıç

(Joint work with Anne Nijsten, Ruud Pellikaan and Alberto Ravagnani)

In this talk, we establish a link between mathematical knot theory and algebraic coding theory. We explain how one can construct a code from a given knot, and thus regard knots as codes. Moreover, we give series of results illustrating how the properties of codes help us determine those of knots via the said constructions.

Algebras, projective planes, incidence geometries, and algebraic curves

Michel Lavrauw

In this talk I will explain the context, motivation, and main results from [BBL] (joint work with Aart and Simeon) and [L], and the more recent classification result from [LR]. I will approach the subject from a historical point of view, starting at 1900, and include some elements from algebra, projective geometry, incidence geometry, and algebraic geometry.

[BBL] S. Ball, A. Blokhuis, M. Lavrauw: On the classification of semifield flocks. Adv. Math. 180 (2003), no. 1, 104–111.

[L] M. Lavrauw: Sublines of prime order contained in the set of internal points of a conic. Des. Codes Cryptogr. 38 (2006), no. 1, 113–123.

[LR] M. Lavrauw, M. Rodgers: Classification of 8-dimensional rank two commutative semifields. Adv. Geom. 19 (2019), no. 1, 57–64.

On the geometry of the Hermitian Veronesean Curve

Stefano Lia

University College Dublin

(Joint work with Michel Lavrauw and Francesco Pavese)

In combinatorics and finite geometry, the study of algebraic groups and their various actions has often led to new constructions of interesting (rare) geometric objects. It is an essential feature of the interplay between groups and geometry. A well-known example, due to Jacques Tits from 1962, is the action of the Suzuki group on the points of a 3-dimensional projective space, giving rise to an ovoid (a notion introduced by Beniamino Segre): a set of points which has the same combinatorial and geometric properties as (but is not equivalent to) an elliptic quadric. Since then, this idea has matured, and the availability of computer algebra systems has greatly contributed to recent developments; there are many authors who have used so-called "orbit-stitching" to obtain new constructions of desirable (finite) geometries. In this talk we will focus on the action of the group of projective motions of certain algebraic varieties. The classification of their orbits on subspaces is a challenging task, and few classifications are complete.In this talk I will focus on the action of an algebraic group $G \leq PGL(4, q)$, isomorphic to $PGL(2, q)$, arising from a maximal rational curve embedded on a smooth Hermitian surface with some fascinating properties. The study of its orbits leads to a new construction of a quasi-Hermitian surface: a set of points with the same combinatorial and geometric properties as a non-degenerate Hermitian surface

A question that involves graph theory, matrix theory, and finite geometry

Marko Orel

University of Primorska, IMFM

Let Γ be a finite simple graph on n vertices, and let $\chi(\Gamma)$, $\omega(\Gamma)$, $\alpha(\Gamma)$ be its chromatic/clique/independence number, respectively. The question $\chi(\Gamma) \overset{?}{>} \omega(\Gamma)$ is often difficult to answer. For several interesting graphs it is equivalent to the question $\alpha(\Gamma)\omega(\Gamma) \overset{?}{<} n$. In the talk I will discuss how these questions intertwine graph theory (study of cores and graph homomorphisms), matrix theory (preserver problems), and finite geometry (the existence of ovoids in finite classical polar spaces).

Some of my favourite problems on projective planes

Eric Moorhouse

University of Wyoming

I will share some of my thoughts about projective planes, including a few constructions and results, but mostly (in the tradition of Erdős talks) open problems.

The extensible No-Three-In-Line problem

Zoltán Lóránt Nagy

Eőtvős University & ELKH-ELTE GAC Research Group

Joint work with Dániel Nagy, Russ Woodroofe

The classical No-Three-In-Line problem seeks the maximum number of points that may be selected from an $n \times n$ grid while avoiding a collinear triple. The maximum is well known to be linear in n, as ovals from finite projective planes over prime order provide examples.
Following a question of Erde, we seek to select sets of large density from the infinite grid $\mathbb{Z}^2$ while avoiding a collinear triple. We show the existence of such a set which contains $\Theta(n/\log^{1+\varepsilon} n)$ points in $[1, n]^2$ for all n, where $\varepsilon > 0$ is an arbitrarily small real number. We also give computational evidence suggesting that a set of lattice points may exist that has at least $n/2$ points on every large enough $n \times n$ grid.

[1] Nagy, D. T., Nagy, Z. L., & Woodroofe, R. (2022). The extensible No-Three-In-Line problem. arXiv preprint arXiv:2209.01447.

Line-free sets

Péter Pál Pach

Budapest University of Technology

In this talk we discuss some bounds about the possible size of sets avoiding certain arithmetic or geometric configurations in $\mathbb{F}_p^n$ (or more generally, in $\mathbb{Z}_m^n$). In particular, we will consider the following forbidden configurations: p-term arithmetic progressions (lines) in $\mathbb{F}_p^3$, right angles in $\mathbb{F}_p^n$ and 6-term arithmetic progressions in $\mathbb{Z}_6^n$.

On relations between Antipodal two-weight rank metric codes and Subspreads of Desarguesian spreads

Tovohery Randrianarisoa

Umeå University

(Joint work with Rakhi Pratihar)

Antipodal two-weight rank metric codes are linear rank metric codes where any non-zero codewords have weight either equal to the minimum rank distance of the code, or equal to the length of the code. In this talk, I explain the relation between antipodal two-weight rank metric codes and the notion of subspreads of Desarguesian spreads. A complete classification of such codes is given when the minimum rank distance is equal to half of the length. In the geometric setting, this says that certain Desarguesian spreads can only have Desarguesian subspreads. We also discuss the problem for other parameters.

Translation Hyperovals in Translation Planes

John Sheekey

University College Dublin

(Joint work with Kevin Allen)

A *hyperoval* in a finite projective plane π of even order q is a set $\mathcal{H}$ of $q+2$ points such that no three points of $\mathcal{H}$ are incident with a common line. Hyperovals can only exist in planes of even order. The study of hyperovals in Desarguesian planes $\mathrm{PG}(2,q)$ has a long history, with various constructions and classifications known.

Hyperovals in general planes have also been considered. It was conjectured that every projective plane of even order contained a hyperoval; this was disproved by the computer classification of Penttila-Royle-Simpson [PRS], where a projective plane of order 16 containing no hyperovals was exhibited.

A *translation plane* is a projective plane with additional structure, and a *translation hyperoval* is a hyperoval with additional structure. Payne [P] showed that all translation hyperovals in $\mathrm{PG}(2,2^n)$ are equivalent to one from a small family of well-understood examples. Cherowitzo [C] computationally classified all hyperovals (translation and otherwise) and their stabilisers in each of the nine translation planes of order 16. In particular he showed that every translation plane of order 16 contains a translation hyperoval, which lead him to conjecture that *every translation plane contains a translation hyperoval*.

In this talk we show that this conjecture is false, by exhibiting a counterexample. We will also highlight connections between this problem and problems in geometry and coding theory, including scattered subspaces with respect to spreads, and the covering radius of rank-metric codes.

[C] Cherowitzo, William E. Hyperovals in the translation planes of order 16. J. Combin. Math. Combin. Comput. 9 (1991), 39–55.

[P] Payne, Stanley E. A complete determination of translation ovoids in finite Desarguian planes. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (8) 51 (1971), 328–331 (1972).

[PRS] Penttila, Tim; Royle, Gordon F.; Simpson, Michael K. Hyperovals in the known projective planes of order 16. J. Combin. Des. 4 (1996), no. 1, 59–65.

Classifying weighted graphs up to Clifford group equivalence

Robin Simoens

Ghent University and Polytechnic University of Catalonia

(Joint work with Simeon Ball)

Let Γ be an undirected, loopless graph on n vertices whose edges have weights in $\mathbb{F}_p$, p prime. Let $A = (a_{ij})_{1 \leq i,j \leq n}$ be its adjacency matrix. Define the following two types of operations:

1. For a given vertex k and for all $i \neq j$ in the neighbourhood of k, add $a_{ik}a_{jk}$ to the weight of the edge connecting them:
$$a_{ij} \mapsto a_{ij} + a_{ik}a_{jk}.$$
2. For a given vertex k and a nonzero $c \in \mathbb{F}_p$, multiply all edges incident with k by c:
$$a_{ik} \mapsto ca_{ik}.$$

We call two graphs Clifford group equivalent if there exists a sequence of these operations that converts one into the other. We are interested in the number of graphs up to this equivalence.

Our motivation comes from stabiliser codes. Two stabiliser codes are equivalent if their stabilisers are the same up to conjugation with a Clifford gate. Such equivalent codes have been shown to correspond with Clifford group equivalent graphs and vice versa.

For $p = 2$ and $n \leq 12$, the number of equivalence classes has been determined. For odd p however, no results are known.

In this talk, I will discuss the above equivalence and present some strategies to compute the number of equivalence classes for $p = 3$. I will explain how this helps us classify the stabiliser codes and certain other quantum error-correcting codes.

Incidence geometries with trialities but without dualities

Klara Stokes

Umeå University

(Joint work with Dimitri Leemans)

Triality is a classical notion in geometry that arose in the context of the Lie groups of type D_4. Another notion of triality, Wilson triality, appears in the context of reflexible maps. We build a bridge between these two notions, showing how to construct an incidence geometry with a triality from a map that admits a Wilson triality. We also extend a result by Jones and Poulton, showing that for every prime power q, the group $L_2(q^3)$ has maps that admit Wilson trialities but no dualities.

[1] D. Leemans and K. Stokes, Incidence geometries with trialities coming from maps with Wilson trialities, arXiv: 2208.08215.

Cameron-Liebler sets in geometrical settings

Leo Storme

Ghent University

Cameron-Liebler sets in finite projective spaces are substructures which can be defined in many equivalent ways; sometimes algebraic, sometimes geometrical. There are even links to Boolean degree one functions [1], [2].

A classical definition of a Cameron-Liebler set of lines $\mathcal{L}$ in PG$(3, q)$ is a set of lines sharing exactly x lines with every spread of PG$(3, q)$.

Research focuses on finding examples of Cameron-Liebler sets with parameter x, or proving that, for a parameter x, Cameron-Liebler sets do not exist.

Examples of non-trivial Cameron-Liebler sets with parameter x in PG$(3, q)$ have been found, but a modular condition, found by Gavrilyuk and Metsch, eliminates the existence of Cameron-Liebler sets in PG$(3, q)$ for more than 50% of the possible parameters x [3].

Cameron-Liebler sets is a topic on which many new results have been found and, hopefully, still will be found.

This talk will focus on different aspects of Cameron-Liebler sets in finite projective spaces, finite affine spaces, and finite classical polar spaces.

[1] A. Blokhuis, M. De Boeck, and J. D'haeseleer, Cameron-Liebler sets of k-spaces in PG(n, q). *Des. Codes Cryptogr.* **87** (2019), no. 8, 1839–1856.

[2] M. De Boeck and J. D'haeseleer, Equivalent definitions for (degree one) Cameron-Liebler classes of generators in finite classical polar spaces. *Discrete Math.* **343** (2020), no. 1, 111642, 13 pp.

[3] A.L. Gavrilyuk and K. Metsch, A modular equality for Cameron-Liebler line classes. *J. Combin. Theory Ser. A* **127** (2014), 224–242.

Some old and new results of Aart in Galois Geometry

Tamas Szőnyi

Generalized Korchmáros-Mazzocca arcs and renitent lines

Zsuzsa Weiner

ELKH-ELTE GAC

(Joint work with Bence Csajbók and Péter Sziklai)

Korchmáros-Mazzocca arcs are point sets of size $q + t$ intersecting each line in 0, 2 or t points in a finite projective plane of order q. When $t \neq 2$, this means that each point of the point set is incident with exactly one t-secant. For $t = 1$, we get the ovals, for $t = 2$ the hyperovals; hence this concept generalizes well-known objects of finite geometry. They were introduced and first studied by Korchmáros and Mazzocca in 1990, see [KM]. In [CsW], with Bence Csajbók, we generalized the concept of Korchmáros-Mazzocca arcs, namely in $\mathrm{PG}(2, q)$, $q = p^h$, we changed 2 in the definition above to any integer m. Also, we introduced the mod p variants of these objects. In this talk, I will give examples and some characterization type results on these objects, for example I will describe all examples[when m or t is not divisible by p. I will also show how all these results relate to some of Aart's and his coauthors' beautiful work [BBM], [BSW], [BSz], [BSzW]). Under some condition, we also proved the existence of a nucleus. In order to do so, we had to show that the 'renitent' lines (the t-secants) through the points of an m-secant have a nucleus (and a similar lemma holds for the mod p variant of the problem). Together with Bence Csajbók and Péter Sziklai ([CsSzW]), we studied possible generalizations of the phenomenon above, i.e. we investigated point sets of a desarguesian affine plane, for which there exist some (sometimes: many) parallel classes of lines, such that almost all lines of one parallel class intersect our set in the same number of points (possibly mod p, the characteristic). We proved results on the (regular) behaviour of the lines with exceptional intersection numbers, which can be viewed as the extension of the ideas in [B] and [BSz]. In this talk, I will also give some insight into this study. As a consequence of these results, I will present a natural generalisation of a nice lemma which helped Blokhuis, Brouwer and Wilbrink to prove that unitals which are codewords are necessarily Hermitian ([BBW]).

[BBM] S. Ball, A. Blokhuis, F. Mazzocca, Maximal arcs in Desarguesian planes of odd order do not exist. *Combinatorica* **17**(1), (1997), 31–41.

[B] A. Blokhuis, Characterization of seminuclear sets in a finite projective plane, *J. Geom.* **40** (1991), 15–19. [BBW] A. Blokhuis, A.E. Brouwer, H. Wilbrink, Hermitian unitals are code words, *Discrete Math.* **97** (1991), 63–68.

[BSW] A. Blokhuis, Á. Seress, H. A. Wilbrink, On sets of points without tangents, Mitt. Math. Sem. Univ. Giessen 201 (1991), 39?44.

[BSz] A. Blokhuis and T. Szőnyi, Note on the structure of semiovals in finite projective planes, *Discrete Math.* **106/107** (1992), 61–65.

[BSzW] A. Blokhuis, T. Szőnyi, Zs. Weiner, On Sets without Tangents in Galois Planes of Even Order, *Designs, Codes and Cryptography* **29** (2003), 91–98.

[CsW] B. Csajbók, Zs. Weiner, Generalizing Korchmáros-Mazzocca arcs, *Combinatorica*, **41** (2021) 601–623.

[CsSzW] B. Csajbók, P. Sziklai, Zs. Weiner, Renitent lines, submitted.

[KM] G. Korchmáros and F. Mazzocca, On $(q+t)$-arcs of type $(0,2,t)$ in a Desarguesian plane of order q. *Math. Proc. Cambridge Philos. Soc.* **108** (3) (1990), 445–459.

23 years of scattered spaces

Ferdinando Zullo

Università degli Studi della Campania “Luigi Vanvitelli”

Since the seminal paper [BL] by Blokhuis and Lavrauw in 2000, scattered spaces have been used to construct or characterize a wide variety of geometrical and algebraic objects, such as translation hyperovals, translation caps in affine spaces, two-intersection sets, blocking sets, translation spreads of the Cayley generalized hexagon, finite semifields, linear codes and graphs, [L]. The past 23 years saw significant progress on scattered spaces and related problems, accompanied by the development of a rich set of new tools. In this talk we will go through this progress, exploiting some recent results. Besides “old” results, we will focus on the recent generalization [GRSZ], [CMPZ] and their applications.

[BL] A. Blokhuis and M. Lavrauw: Scattered spaces with respect to a spread in $\mathrm{PG}(n, q)$, *Geom. Dedicata* **81** (2000), 231–243.

[CMPZ] B. Csajbók, G. Marino, O. Polverino and F. Zullo: Generalising the scattered property of subspaces, *Combinatorica* **41(2)** (2021), 237–262.

[GRSZ] A. Gruica, A. Ravagnani, J. Sheekey and F. Zullo: Generalized scattered spaces, arXiv:2207.01027.

[L] M. Lavrauw: Scattered spaces in Galois geometry, *Contemporary Developments in Finite Fields and Applications*, 2016, 195–216.