Lectures 12

Type system

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Literature

- John Mitchell, Concepts in Programming Languages, Cambridge Univ Press, 2003 (Chapter 6)
- Michael L. Scott, Programming Language Pragmatics (3rd ed.), Elsevier, 2009 (Chapter 7)
- Emmanuel Chailloux, Pascal Manoury, Bruno Pagano, Developing Applications With Objective Caml, O’REILLY & Associates, 2000

Outline

1. Introduction
2. Type equivalence
3. Type casts
4. Type compatibility
5. Coercions
6. Typing rules
7. Type inference

Introduction

- At least three ways to think about types
- Denotational
  - Type is simply a set of values
  - Value has a given type if it belongs to the set
- Constructive
  - Type is either one of a small collection of built-in types
  - Created by applying a type constructor (record , array , set, etc.) to one or more simpler types
- Abstraction-based
  - Type is a data structure
  - Type is an interface consisting of a set of operations with well-defined semantics

Why types?

- Naming and organizing concepts
  - Structuring data
  - Documenting data organization
- Consistent interpretation of data (bit sequences) in computer memory
  - Type errors
- Providing information to the compiler about data manipulated by the program
  - Memory layout for accessing data
  - Compatibility of operation operands
  - Locating references for garbage collection

Why types?

There are two basic functions of types:

1. Types provide implicit context for many operations, so that the programmer does not have to specify that context explicitly
   - a + b, new p, ...
   - Choice of operation, sizes of structures,
2. Types limit the set of operations that may be performed in a semantically valid program
   - Expressions and values with attached “meaning”
   - Typing and then catching type errors significantly improves code
   - Catch nonsensical operation (1+”banana”)

Type systems

- A type system consists of:
  1. A mechanism to define types and associate them with certain language constructs
  2. A set of rules for type equivalence, type compatibility, and type inference
  3. A type-checking algorithm
- Which constructs have types?
  - Imperative language: those that have values
    - named constants, variables, record fields, parameters, subroutines, expressions
  - Functional languages: any expression has a type
    - Functions, values, expressions, statements, classes, modules

Two kinds of type systems

- Two lambda calculus with types:
  - Implicit types, Curry Haskell
  - Explicit types, Alonzo Church
- Implicit types
  - Or, Curry type annotations
  - Optional type annotations
  - Type annotations are added where needed
  - Types are derived from expressions
  - Sophisticated type inference algorithms
  - ML, Haskell, Ocaml
    - Functional languages (not Lisp)

Two kinds of type systems

- Explicit types
  - Or, Church type annotations
  - Strict type annotations
  - Language implementations include verification of types of variables, expressions, etc.
  - Types derived from expressions must be equivalent to annotations
  - Imperative languages usually use explicit type annotations
  - Pascal, C, C++, Java, Scala

Type systems

- Type equivalence rules
  - Determine when the types of two values are the same
- Type compatibility rules
  - Determine when a value of a given type can be used in a given context
- Type inference rules
  - Define the type of an expression based on the types of its constituent parts or the surrounding context
- Type-checking procedure
  - Given a program, checks all expressions that have types by using type equivalence, compatibility and type inference

Type checking

- When an object of a certain type can be used in a certain context?
- At this point the following three procedures are needed to judge the position
  - Type equivalence and/or compatibility
  - Type inference
- At a minimum, the object can be used if its type and the type expected by the context are equivalent
  - Compatibility is a looser relationship than equivalence
  - Objects and contexts are often compatible even when their types are different

Type checking

- Type compatibility is the one of most concern to programmers
  - Type compatibility can involve: type conversion (cast), coercion
- Type inference procedure computes type of an expression constructed from simpler subexpressions
  - Given the types of the sub-expressions (and possibly the type expected by the surrounding context), what is the type of the expression as a whole?

Type checking

- Another view of type checking
  - Type checking is the process of ensuring that a program obeys the language's type compatibility rules
- A language is strongly typed
  - Prohibits the application of any operation to any object that is not intended to support that operation
- A language is said to be statically typed
  - Strongly typed? Yes / **No**.
  - Most type checking can be performed at compile time
  - Ada, Pascal, C, C++, Java, Scala

Type checking

- Dynamic (run-time) type checking
    - A form of late binding
    - Tends to be found in languages that delay other issues until run time as well
    - Lisp, Smalltalk are dynamically typed
    - Most scripting languages are dynamically typed
    - Python, Ruby are also strongly typed

Type checking and polymorphism

- Polymorphism
  - Single body of code works with objects of multiple types
    - It may or may not imply the need for run-time type checking
- Dynamic typing
  - Supports implicit parametric polymorphism
    - Types can be thought of as implied (unspecified) parameters
    - Types of arguments are checked in run-time
  - Powerful and straightforward
    - Operation implementation is selected at run-time
  - Languages Lisp, Smalltalk, etc.
  - Significant run-time cost for type checking

Type checking and polymorphism

- Subtype polymorphism in OO languages
  - Given a straightforward model of inheritance
  - Type checking for subtype polymorphism can be implemented entirely at compile time.
- Explicit parametric polymorphism
  - A class is specified by using type parameters
  - Generics in C++, Eiffel, Java, and C#
  - Useful as a base class for the containers
  - Compile-time static type checking suffices
    - Similarly to subtype polymorphism

Type checking and polymorphism

- ML family
  - Sophisticated system of type inference
  - ML compiler infers for every expression a type
    - With rare exceptions, the programmer need not specify the types of objects explicitly
  - Task of the compiler is to determine whether there exists a consistent assignment of types to expressions
    - This guarantees, statically, that no operation will be applied to a value of an inappropriate type at run time
    - Formalized as the problem of unification
  - Implicit parametric polymorphism with static typing
    - Computes the most general types
    - Derives type variables if there is no other constraints

Type equivalence

- Two principal ways of defining type equivalence
- Structural equivalence is based on content of type definitions
  - Two types are the same if they consist of the same components
  - Algol-68, Modula-3, C and ML
- Name equivalence is based on the lexical occurrence of type definitions
  - More popular approach in recent languages
  - Java, C#, standard Pascal, and most Pascal descendants, including Ada

Structural equivalence

- Exact definition varies from one language to another
  - ML says Ok; most languages say error
- Two types are structurally equivalent
  - Replace any embedded type names with their definitions, recursively
  - Until nothing is left but type constructors, field names, and built-in types
  - Then, compare structures
  - Problem is an inability to distinguish between types that the programmer may think of as distinct

```
/* Pascal */
type R2 = record
  a, b : integer
end;
/* same as? */
type R3 = record
  a : integer;
  b : integer
end;
/* what about this? */
type R4 = record
  b : integer;
  a : integer
end;
```

```
type student = record
  name, address : string
  age : integer
type school = record
  name, address : string
  age : integer
x : student;
y : school;
...
x := y;   /* ? */
```

Name equivalence

```
type student = record
   name, address : string
   age : integer
type school = record
   name, address : string
   age : integer
x : student;
y : school;
…
x := y;   /* ? */
```

- Assumption:
  - If programmer writes two definitions (for the same type) then they are meant to represent different types
- Example:
  - Variables x and y are of different type and (under name equivalence) therefore we have type-checking error
- Name equality means that two type names are considered equal in type checking only if they are the same

Variants of name equivalence

- There are two variants of name equivalence
- The simplest of type definitions

```
TYPE new_type = old_type;    /* Modula-2*/
```

- Here new_type is said to be an alias for old_type
  - Should we treat them as two different names or the names of the same type?
- Strong name equivalence
  - Treat them strictly as different types
- Loose name equivalence
  - Treat them as two names of the one type

```
TYPE celsius_temp = REAL;
        fahrenheit_temp = REAL;
VAR c : celsius_temp;
        f : fahrenheit_temp;
...
f := c;  /* error? */
```

```
TYPE stack_element = INTEGER;  (* alias *)
```

Type conversion (type cast)

- Explicit type conversion!
- There are many contexts in which values of a specific type are expected

```
a := expression
```

  - We expect right-hand side to have the same type as a

```
a + b
```

  - The overloaded + symbol designates either integer or floating-point addition
    - both integers or both reals

```
foo(arg1, arg2, . . . , argN)
```

  - We expect the types of the arguments to match those of the formal parameters
- Suppose in each of these cases that the types (expected and provided) are exactly the same

Type conversion (type cast)

- To use a value of one type in a context that expects another we can use explicit type conversion (or, type cast)
  1. Types employ the same low-level representation, and have the same set of values
     - No code will need to be executed at run time
  2. Types have different sets of values, but the intersecting values are represented in the same way
     - One type may be a subrange of the other
     - Run-time check of exact types; can generate run-time error
  3. Types have different low-level representations but there is correspondence among the values
     - integer ↔ floating-point

Example

```
type test_score = 0..100;
        workday = mon..fri;
type celsius_temp is new integer;
type fahrenheit_temp is new integer;
```

```
/* Ada */
n : integer;              -- assume 32 bits
r : real;                 -- assume IEEE double-precision
t : test_score;           -- as in Example 7.9
c : celsius_temp;         -- as in Example 7.20
...
t := test_score(n);       -- run-time semantic check required
n := integer(t);          -- no check req.; every test_score is an int
r := real(n);             -- requires run-time conversion
n := integer(r);          -- requires run-time conversion and check
n := integer(c);          -- no run-time code required
c := celsius_temp(n);     -- no run-time code required
```

Type Compatibility

- Most languages do not require equivalence of types in every context
- Value’s type must be compatible with that of the context in which it appears
    - Left and right side of assignment statement
    - Values used in arithmetic operations
    - Actual parameter in function call
- The definition of type compatibility varies greatly from language to language

Coercion

- Language allows a value of one type to be used in a context that expects another
  - Language implements automatic, implicit conversion to the expected type!
  - Run-time code must perform a dynamic semantic check, or convert between low-level representations
- OCaml provides explicit coercion
  - Coercion operator ":>"
  - Programmer has to take care of conversions
    - Avoiding errors that are hard to find
  - Base types and objects can be coerced
  - Separate operations for separate types (+, +., ...)
  - We will see more in chapter on OO languages

(name : sub type :> super type)

(name :> super type)

Coercion

- C++ provides an extremely rich, programmer-extensible set of coercion rules
  - Coercion code can be defined when new type is defined
  - This makes C++ flexible
  - One of the most difficult C++ features to understand and use correctly
  - Rules interact in complicated ways with the rules for resolving overloading

Coercion in C

- C performs quite a bit of coercion

```
short int s;
unsigned long int l;
char c;     /* may be signed or unsigned -- implementation-dependent */
float f;      /* usually IEEE single-precision */
double d;  /* usually IEEE double-precision */
...
s = l; /* l’s low-order bits are interpreted as a signed number. */
l = s; /* s is sign-extended to the longer length, then
            its bits are interpreted as an unsigned number. */
s = c; /* c is either sign-extended or zero-extended to s’s length;
            the result is then interpreted as a signed number. */
f = l;   /* l is converted to floating-point. Since f has fewer
              significant bits, some precision may be lost. */
d = f;  /* f is converted to the longer format; no precision lost. */
f = d;  /* d is converted to the shorter format; precision may be lost.
              If d’s value cannot be represented in single-precision, the
              result is undefined, but NOT a dynamic semantic error. */
```

Coercion in Fortran

- Fortran allows arrays and records to be intermixed if their types have the same shape
- Two arrays are of the same shape
  - The same number of dimensions, elements and the same shape of element type
- Two records have the same shape
  - The same number of fields, and the fields are of the same shape
  - Field names do not matter, nor do the actual high and low bounds of array dimensions

Trends in coercion use

- Modern compiled languages display a trend toward static typing and away from type coercion
- Some language designers argue that coercions are a natural way in which to support abstraction and extensibility
  - It is easier to use new types together with existing ones
  - This is especially true for scripting languages

Type inference

- Type inference is used for type-checking
  - The process of determining the types of expressions based on the known types
  - Inferred types are compared to types expected in a given context
- There are two general approaches to type inference:
  1) Type inference algorithms is based on typing rules that derive concrete (ground) types
     - Pascal, Java, C, C++,
  2) Type inference algorithms based on typing rules that derive parametrized types
     - ML, Ocaml, Haskell

Type inference based on rules

- Type of an expression is inferred by means of typing rules
- During compilation expression is parsed into abstract syntax tree (AST)
  - AST is used to attach the type to each of sub-expressions
    - Check the lecture on Compilers and interpreters
- Types are computed bottom-up
  - A type of an expression is computed from types of its sub-expressions
  - Typing rules act as patterns that match given syntactic constructions

Typing rules

- Typing rules concern judgments of the form

$$\Gamma \vdash e : T$$

  - where Γ is a context, which contains e.g. typings of identifiers
  - The judgment says: in the environment Γ, expression e has type T
- Judgments are used in typing rules of the form

$$\frac{J_1 \quad J_2 \quad \dots \quad J_n}{J} \quad C$$

  - J_i are called premises, J is called conclusion and C condition

Typing rules

- Example rule:
  - If x and y have type int then x+y has type int

$\Gamma = x{:}int, y{:}int$

$$\frac{\Gamma \vdash x{:}int \qquad \Gamma \vdash y{:}int}{\Gamma \vdash x + y : int}$$

- Context Γ is written in the form

$$\Gamma = x_1{:}T_1,x_2{:}T_2...,x_n{:}T_n$$

- Judgement form for typing is generalized to

$$\Gamma \vdash e{:}T$$

- To add a new variable to the context Γ, we write

$$\Gamma, x : T$$

Example

- Type checking rules for arithmetic expressions

$$\frac{\Gamma \vdash e1 : \text{int} \quad \Gamma \vdash e2 : \text{int}}{\Gamma \vdash e1 + e2 : \text{int}} \qquad \frac{\Gamma \vdash e1 : \text{int} \quad \Gamma \vdash e2 : \text{int}}{\Gamma \vdash e1 * e2 : \text{int}}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \qquad \frac{}{\Gamma \vdash i : \text{int}} \text{ i is an integer literal}$$

- Derivation of judgment: x : int, y : int => x + 12 * y : int

$$\dfrac{x:\text{int}, y:\text{int} \vdash x : \text{int} \qquad \dfrac{\text{(*int literal*) } x:\text{int}, y:\text{int} \vdash 12 : \text{int} \quad x:\text{int}, y:\text{int} \vdash y : \text{int}}{x:\text{int}, y:\text{int} \vdash 12 * y : \text{int}}}{x:\text{int}, y:\text{int} \vdash x + 12 * y : \text{int}}$$

Typing functions

- Type of function with one parameter is written

$$f : T_1 \to T_2$$

- The typing rule for function says: if x has type T_1 and f has type $T_1 \to T_2$, then f(x) has type T_2

$$\frac{\Gamma \vdash x:T_1 \qquad \Gamma \vdash f : T_1 \to T_2}{\Gamma \vdash f(x) : T_2}$$

- Typing rule for functions with more than one parameters (one parameter seen as tuple)

$$f : T_1 * \ldots * T_n \to T$$

- Expression “$f : T_1 * \ldots * T_n \to T$” is called signature

Typed λ-calculus

$$\frac{x:T \in \Gamma}{\Gamma \vdash x:T} \quad \text{(axiom)}$$

$$\frac{\Gamma \vdash M : (\sigma \to \tau) \qquad \Gamma \vdash N : \sigma}{\Gamma \vdash (M\ N) : \tau} \quad (\to\text{-elimination})$$

$$\frac{\Gamma, x : \sigma \vdash M : \tau}{\Gamma \vdash (\lambda x.M) : (\sigma \to \tau)} \quad (\to\text{-introduction})$$

Type inference in ML

- ML type inference algorithm derives most general parametrized type of expression
  - H. Curry, R. Feys, R. Hindley, R. Milner
    - Hindley-Milner type system
  - Type inference can be applied to a variety of programming languages
- ML type inference supports polymorphism
  - Type variables are used as place-holders for types that are not known
- Algorithm will be presented by examples

Example 1

```
– fun f1(x) = x + 2;
val f1 = fn : int → int
```

- Type of 2 is int
- Operator + is overloaded but since we have one integer, then it must have type int→(int→int)
- Therefore, x must be of type int
- Putting this together we get that f1 is of type int→int

Example 2

```
– fun f2(g,h) = g(h(0));
val f2 = fn : (’a → ’b) * (int → ’a) → ’b
```

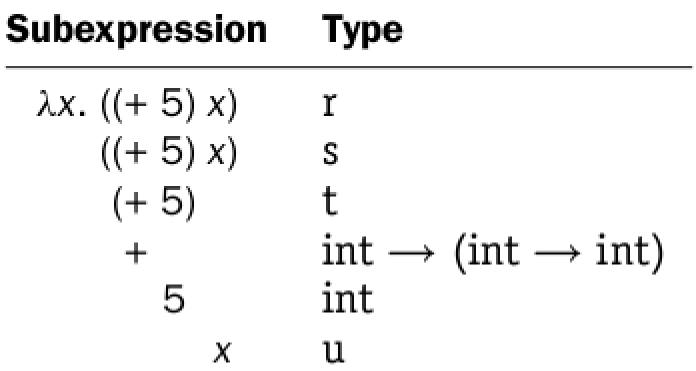
- Type of 0 is int
- The type of function h result is not known, therefore we write 'a
- Since the result of h is an argument of g then the domain of g is 'a
- Also the type of g is not known so we take 'b
- Since type of g result is 'b then also result of f2 is of type 'b
- We get the type ('a→'b)*(int→'a)→'b

Type-Inference Algorithm

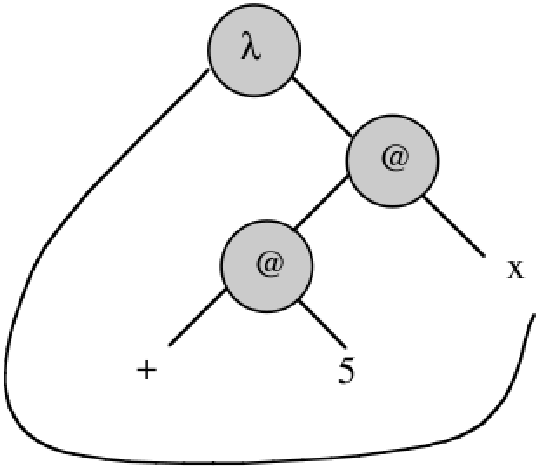
1. Assign a type to the expression and each subexpression
    - For any compound expression or variable, use a type variable
2. Generate a set of constraints on types, using the parse tree of the expression
3. Solve these constraints by means of unification, which is a substitution-based algorithm for solving systems of equations

Example 3

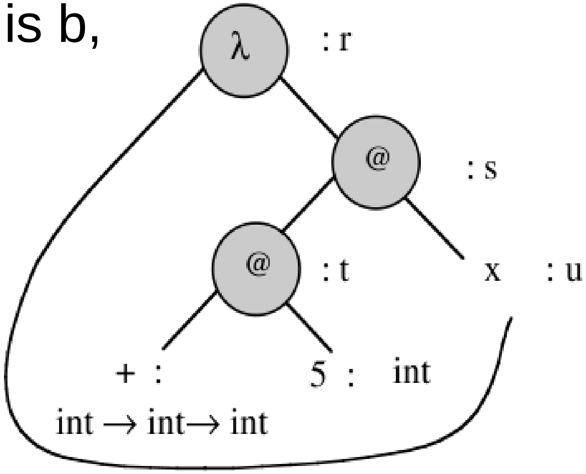
```
- fun g(x) = 5 + x;
val g = fn : int → int
```



Subexpression	Type
$\lambda x.\ ((+\ 5)\ x)$	r
$((+\ 5)\ x)$	s
$(+\ 5)$	t
+	$\text{int} \longrightarrow (\text{int} \longrightarrow \text{int})$
5	int
x	u



(1)



Function Application: If the type of f is a, the type of e is b, and the type of f(e) is c, then we must have $a = b \to c$.

Lambda Abstraction: If the type of x is a, the type of e is b, and the type of λx.e is c, then we have $c = a \to b$.

(2) We get constraints:

Subexpression (+5): $\text{int} \to (\text{int} \to \text{int}) = \text{int} \to t$

Subexpression (+5) x: $t = u \to s$

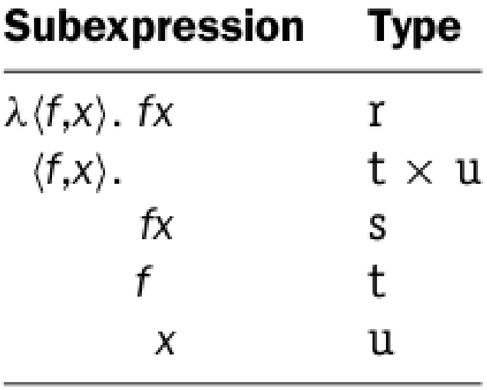
Subexpression λx.((+5)x): $r = u \to s$

(3) Solve equations

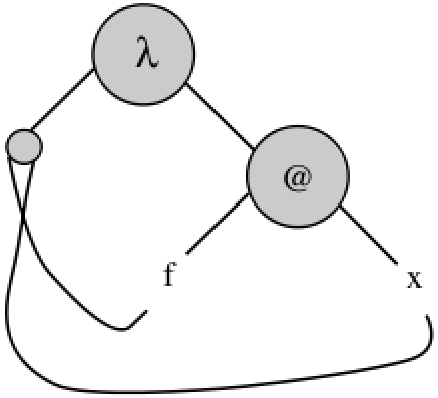
$t=\text{int}\to\text{int}$, $u=\text{int}$, $s=\text{int}$, $r=\text{int}\to\text{int}$

Example 4

```
– fun apply(f,x) = f(x);
val apply = fn : (’a → ’b) * ’a → ’b
```



Subexpression	**Type**
$\lambda\langle f,x\rangle.\ fx$	r
$\langle f,x\rangle.$	$t \times u$
fx	s
f	t
x	u



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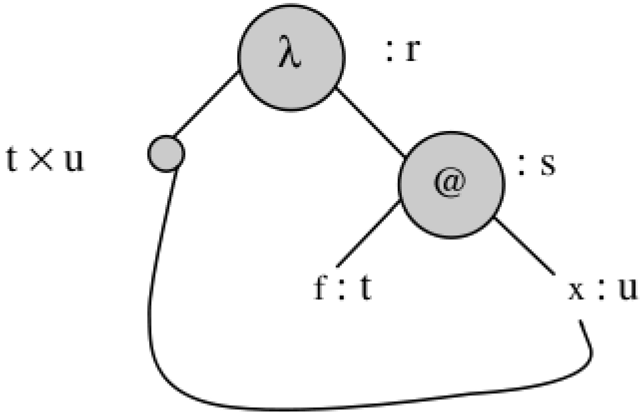

(2) Generate constraints:

$t=u\to s$

$r=t*u\to s$

(3) Solve constraints:

$r=(u\to s)*u\to s$

$r=('a\to 'b)*'a\to 'b$