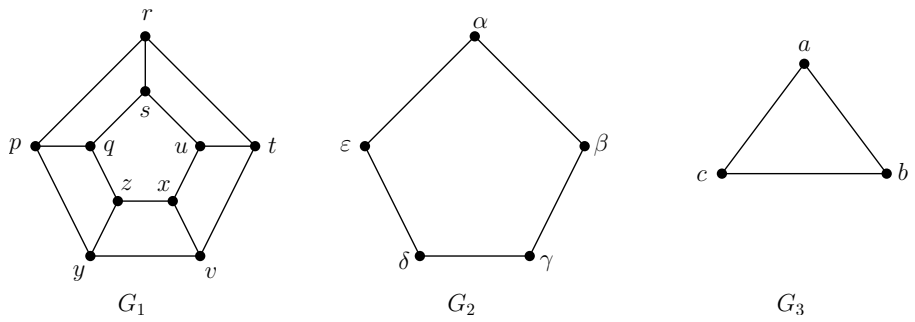
17 Graph Homomorphisms

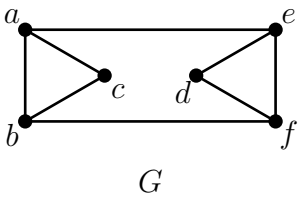
95. Let $G = (V, E)$ and $G' = (V', E')$ be (simple) graphs. We say that a graph G is **homomorphic** to a graph G' if there exists a mapping $\phi : V \to V'$ such that for every edge $\{x, y\} \in E$ of graph G, it holds that $\{\phi(x), \phi(y)\} \in E'$. Such a mapping ϕ is called a **homomorphism** of graph G to graph G' and is denoted as $\phi : G \to G'$.

Show that for any three graphs G_1, G_2, and G_3, if there exists a homomorphism $f : G_1 \to G_2$ and a homomorphism $g : G_2 \to G_3$, then there also exists a homomorphism $h : G_1 \to G_3$. Using the proof, find the corresponding homomorphisms for the following graphs:



18 Induced Subgraph, Graph Isomorphisms, and Closed Walks

96. We are given a graph G:



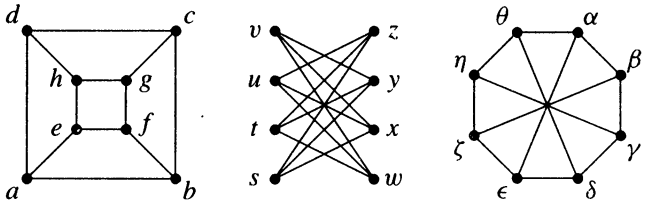
(a) For the graph G in the figure, provide an example of an induced subgraph that is not bipartite and has 5 vertices.

(b) Find four isomorphisms of graph G onto itself (i.e., graph G onto itself). (An isomorphism of graph G onto itself is called an *automorphism* of graph G).

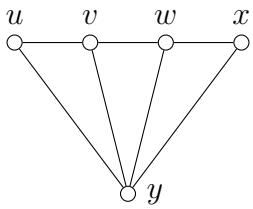
(c) For the graph G in the figure, provide an example of a closed walk that includes all edges of the graph, or explain why such a walk does not exist.

19 Graph Isomorphisms

97. Determine which pairs of graphs are isomorphic.



98. For the graph G in the figure:



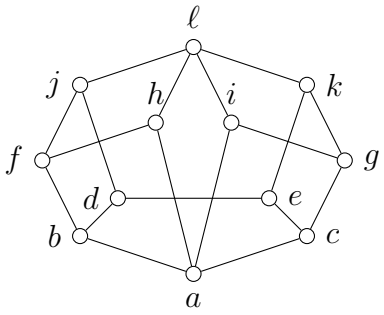
G

(a) Provide examples of three pairs of non-isomorphic spanning trees.

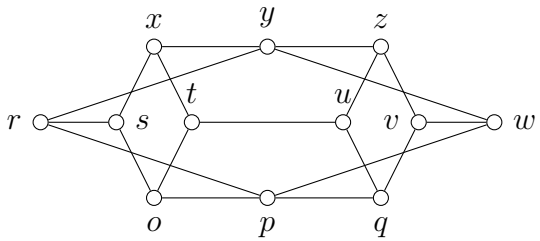
(b) Provide examples of three pairs of non-isomorphic spanning subgraphs that are not bipartite.

Justify your answer precisely.

99. We are given the following graphs:



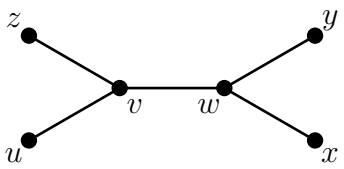
G



H

Are the graphs G and H isomorphic? Provide a precise justification for your answer.

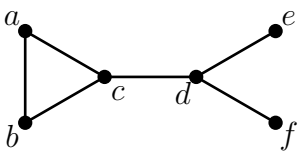
100. We are given a graph G:



G

Find all isomorphisms of graph G onto itself (i.e., graph G onto itself). (An isomorphism of graph G onto itself is called an *automorphism* of graph G). Provide a precise justification for your answer.

101. We are given a graph G:



G

Find all isomorphisms of the graph G onto the graph G (i.e., the graph G onto itself). (An isomorphism of the graph G onto itself is called an *automorphism* of the graph G).

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
The reader can find all solutions to the given problems on the same page.