13 Distributions

68. (a) Write down all the possibilities for arranging three different items (apple, orange, and banana) into two *distinct* boxes so that no box is empty.

(b) Let n be a natural number. Provide a combinatorial proof of the following equation:

$$S(n, 2) = 2^{n-1} - 1.$$

69. In how many ways can we arrange 8 different rings into 3 boxes,

(a) if the boxes are identical and each box must contain at least one ring?

(b) if the boxes are distinct and each box must contain at least one ring?

Provide the result as an exact numerical value. Before computing the numerical values, list at least three examples of possible arrangements.

70. In how many ways can 10 indistinguishable balls be placed into eight distinguishable containers?

71. In how many ways can four different employees be assigned to three indistinguishable offices if each office can accommodate any number of employees?

72. Four fishermen went to a lake to catch fish. At the end of the day, they collectively caught 11 identical fish. They now need to divide these 11 fish into 4 identical bags. In how many ways can this be done if

(a) each bag contains at least one fish?

(b) some bags can be empty?

Provide the results as exact numerical values.

73. Four fishermen went to a lake to catch fish. At the end of the day, they collectively caught 13 identical fish. They now need to divide these fish among themselves. In how many ways can this be done if:

(a) each fisherman must receive at least one fish?

(b) some fishermen can receive no fish?

Provide the results as exact numerical values.

74. We need to assign 6 distinct household chores to 5 workers such that each worker gets at least one chore. In how many ways can this be done?

75. A standard deck of cards has 52 distinct cards. Each of the four suits (spades, clubs, hearts, diamonds) contains 13 cards, ranging from Ace (the lowest) to King (the highest). In how many ways can all the cards be distributed among 4 players such that each player receives at least one card?

76. In how many ways can eight DVDs be packed into three boxes such that each box contains at least one DVD, if we have

(a) eight different DVDs and three indistinguishable boxes,

(b) eight different DVDs and three distinguishable boxes?

Provide the results as exact numerical values.

77. In how many ways can eight DVDs be packed into three boxes such that each box contains at least one DVD, if we have

(a) eight different DVDs and three distinguishable boxes,

(b) eight indistinguishable DVDs and three indistinguishable boxes,

(c) eight indistinguishable DVDs and three distinguishable boxes?

Provide the results as exact numerical values.

78. In how many ways can seven different balls be distributed into four indistinguishable boxes if each box can contain any number of balls?
There is no need to provide the result as an exact numerical value.

79. In how many ways can four different employees be assigned to three indistinguishable offices if each office can accommodate any number of employees?

80. In 4 distinct art classes, we enroll 12 children. Among the children, there are two pairs of siblings, and siblings are not to be placed in separate groups. Count the different ways to do this, ensuring that no class is empty.

81. (a) We want to divide 12 children into four playgroups. However, there are **exactly two pairs** of siblings among the children, and siblings are not to be placed in separate groups. How many possibilities do we have?

(b) In how many ways can 30 mangoes be divided into three identical baskets? Some baskets may remain empty.

Provide exact numerical values for your results.

82. In how many ways can 9 rings be distributed among 4 fingers of the right hand (excluding the thumb), if:

(a) the rings are indistinguishable, and the fingers can also be empty?

(b) the rings are indistinguishable, and each finger must have at least one ring?

(c) the rings are distinct, the order of rings on a finger does not matter, and the fingers can also be empty?

(d) the rings are distinct, the order of rings on a finger does not matter, and each finger must have at least one ring?

(e) the rings are distinct, the order of rings on a finger matters, and the fingers can also be empty?

(f) the rings are distinct, the order of rings on a finger matters, and each finger must have at least one ring?

Provide the result as an exact numerical value. Before calculating the numerical values, list at least three examples of possible arrangements.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
The reader can find all solutions to the given problems on the same page.