11 Linear Recursive Equations with Constant Coefficients

54. Solve the recurrence equation $F(n) = 4F(n-2)$ with initial conditions $F(0) = 0$ and $F(1) = 4$.

55. Solve the recurrence equation $F(n) = 4F(n-1) - 3F(n-2)$ with initial conditions $F(0) = 2$ and $F(1) = 5$.

56. Solve the recurrence equation

$$F(n) = F(n-1) + F(n-2) - F(n-3)$$

with initial conditions $F(0) = F(1) = 0$ and $F(2) = 4$. Use the characteristic equation method.

57. **Lucas Sequence** is defined as $L(0) = 2$, $L(1) = 1$, and $L(n) = L(n-1) + L(n-2)$ for $n \geq 2$.

(a) Compute the first six terms of the sequence.

(b) Derive an explicit (non-recursive) formula for $L(n)$.

12 Recursive Equations

58. (i) Alja deposits 1000€ in a local bank at an annual interest rate of 8%. Define recursively the amount a_n in her account at the end of n years.

(ii) In a meeting room, n guests are present. Each person shakes hands with every other person exactly once. Define recursively the number of handshakes $h(n)$ that occur, and solve the obtained recurrence equation.

59. (i) A bus driver pays a toll using only 1€ and 2€ coins, inserting one coin at a time into a toll machine. Find a recurrence relation for the number of distinct ways the driver can pay a toll of n€ (considering the order of coins).

(ii) Solve the recurrence equation $F(n) = 8F(n-1) - 16F(n-2)$ with initial conditions $F(0) = -1$ and $F(1) = 4$. Use the characteristic equation method.

60. We have L-shaped tiles that cover 3 squares. We want to tile rectangular floors of size $2 \times n$ squares for $n \geq 0$. Find a recurrence relation for the number of ways to tile a $2 \times n$ floor for $n \geq 2$. Determine the numerical value of the number of ways to tile a 2×12 floor.

61. Let a_n be the number of all strings of length n consisting of the letters A, B, and C, but not containing the substring "CA". Find a recurrence relation for a_n. Also, calculate the numerical value of a_5.

62. Let a_n denote the number of binary sequences of length n (sequences of length n containing only digits 0 and 1) that do not contain the subsequence 001.
Find a recurrence relation for a_n and the initial values.
Also, determine the numerical value of a_6.

63. In how many ways can we tile a rectangular floor of size $2 \times n$ squares, $n > 0$, using rectangular tiles of size 1×2 and 2×2 squares? Find a recurrence relation and solve the obtained equation. How many ways are there to tile a floor of size 2×5 squares?

64. Frogs and storks are grazing by the edge of a pond. Each minute, the following happens: in the first half of the minute, each frog present at the beginning of the minute attracts three new frogs and one new stork. In the second half of the minute, each stork eats one frog. Initially, there are two frogs and one stork. Write a recurrence relation that gives the number of frogs and storks at the beginning of the n-th minute for all $n \in \mathbb{Z}_0^+$.

65. We have square tiles of size 1×1 and rectangular tiles of size 1×2. We want to tile a rectangular floor of size $2 \times n$ squares for $n \geq 0$, such that the longer edges of two parallel rectangular tiles must align completely, i.e., the following arrangement is not allowed:

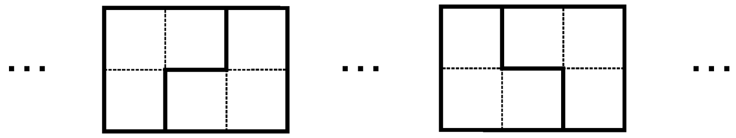
Let $F(n)$ denote the number of different tilings for a $2 \times n$ floor.

(a) Justify that the equation $F(n) = 2F(n-1) + 3F(n-2)$ gives a recurrence relation for the number of different tilings for $n \geq 2$.

(b) Write the initial conditions and verify your result by listing all possible tilings for a floor of size 2×2 squares and comparing it with the result obtained using the recurrence relation.

(c) Provide an explicit equation for $F(n)$.

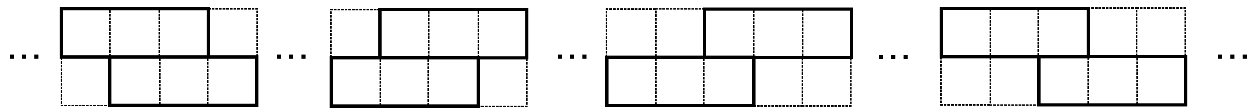
66. We have square tiles of size 1×1 and L-shaped tiles (covering 3 squares). We want to tile a rectangular floor of size $2 \times n$ squares for $n \geq 0$, but the following arrangement is not allowed:



(a) Find a recurrence relation for the number of ways to tile a $2 \times n$ floor for $n \geq 2$.

(b) Write the initial conditions and verify your result by listing all possible tilings for a floor of size 2×2 squares and comparing it with the result obtained using the recurrence relation.

67. We have square tiles of size 1×1 and I-shaped tiles (covering 3 squares). We want to tile a rectangular floor of size $2 \times n$ squares for $n \geq 0$, but the following arrangements are not allowed:



Let $F(n)$ denote the number of different tilings for a $2 \times n$ floor.

(a) Find a recurrence relation for $F(n)$ for $n \geq 3$.

(b) Calculate $F(6)$.

All above math problems are taken from the following website: https://osebje.famnit.upr.si/~penjic/teaching.html.
THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.