

## 9 Unordered Selection with Repetition

**46.** In how many ways can 6 pieces of fruit be selected from a bowl containing apples, oranges, and pears, if the order in which the pieces are selected does not matter and only the type of fruit (not the individual piece) is significant? Assume that the bowl contains at least six pieces of each type of fruit.

**47.** In how many ways can five banknotes be selected from a cash register containing banknotes of denominations €5, €10, €20, €50, €100, €200, and €500? Assume that the order of selection does not matter, banknotes of the same denomination are indistinguishable, and there are at least five banknotes of each denomination. Provide the result as an exact numerical value.

**48.** A group of 40 people has been trained as astronauts for the first mission to Mars. They are divided into 5 groups (red, yellow, blue, green, brown). Each group has 8 different members, each with a training level from 1 to 8. The training level represents the type of training, and each member is trained in exactly one of 8 different tasks (no two members of the same group share the same training level).

- (a) In how many different ways can 6 people be selected such that exactly three training levels are represented, each appearing twice? Provide the result as an exact numerical value.
- (b) In how many ways can 5 people be selected such that they have 5 consecutive training levels? Provide the result as an exact numerical value.

## 10 Unordered Selections without Repetition

**49.** How many integer solutions does the equation  $x_1 + x_2 + x_3 + x_4 = 12$  have if:

- (a)  $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$ ?
- (b)  $x_1 \geq 1, x_2 \geq 2, x_3 \geq 1, x_4 \geq 2$ ?

Provide the result as an exact numerical value. In both parts of the task, give at least three solutions to the given equation before calculating the numerical value.

**50.** How many integer solutions does the equation  $x_1 + x_2 + x_3 + x_4 + x_5 = 22$  have if:

- (a)  $x_1 \geq 0, x_2 \geq 20, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$ ?
- (b)  $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 3, x_5 \geq 2$ ?

Provide the result as an exact numerical value. In both parts of the task, give at least three solutions to the given equation before calculating the numerical value.

**51.** In how many ways can 33 identical drops be distributed into boxes  $A$ ,  $B$ ,  $C$ , and  $D$  if box  $A$  contains at least two drops, box  $B$  contains at least five drops, and box  $C$  contains *exactly* six drops? (Note that there are no restrictions for box  $D$ .) Provide the answer as an exact numerical value.

**52.** A store sells three types of batteries: AAA, AA, and 9V. There are only two 9V batteries left in stock, but more than 10 batteries of each of the other two types. Petra wants to buy 10 batteries and needs at least two AAA batteries. In how many ways can she do this? Calculate the exact numerical value of your solution!

**53.** Sara and Matevž are collecting stones and shells on the beach, gathering a total of 12. There are 4 types: sharp stones, round stones, ribbed shells, and spiral shells. If stones or shells of the same type are indistinguishable, how many collections of size 12 can be made:

- (i) without any additional conditions?
- (ii) if they want at least 4 ribbed shells and 2 spiral shells?
- (iii) if they want at least one stone or shell of each type and at most 3 round stones?

Finally, they collected 1 round stone, 2 sharp stones, 4 ribbed shells, and 5 spiral shells (all colorful and very different) and now wish to divide the loot. Sara wants to keep 2 stones (round or sharp) and 4 shells (ribbed or spiral), while Matevž will take the rest.

- (iv) How many different divisions can they make?

All above math problems are taken from the following website:

<https://osebje.famnit.upr.si/~penjic/teaching.html>.

THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.