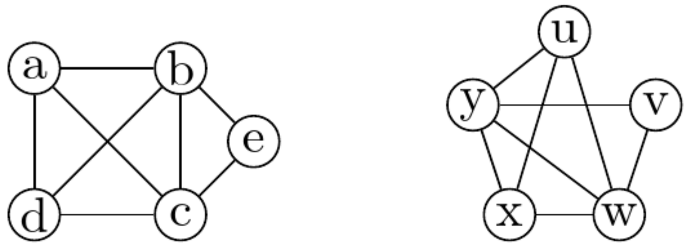
25 Planar Graphs and the Chromatic Number of a Graph

120. Consider the following two graphs:

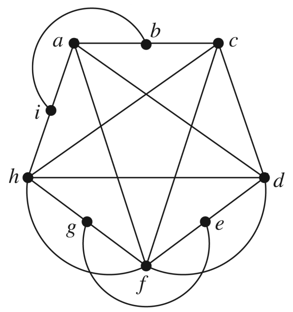
(i) Are they isomorphic to each other?

(ii) Are they planar?



G_1 G_2

121. We are given a graph G:

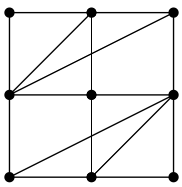


(a) For the graph G in the figure, provide an example of an induced subgraph that is not bipartite and has 5 vertices.

(b) Is G planar?

(c) Determine the chromatic number of the graph G.

122. We are given a graph L:



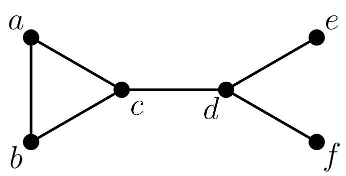
L

(a) Is the graph L planar? Justify your answer.

(b) Find the chromatic number of the graph L.

(c) Is $\chi(L, 2) > 0$? Is $\chi(L, 3) > 0$? Is $\chi(L, 4) > 0$? Justify your answers.

123. Consider the following graph G:

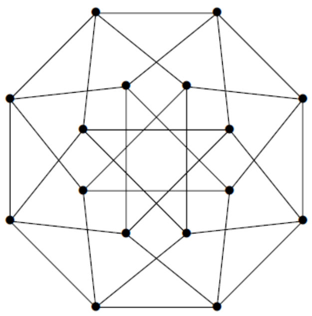


G

(a) Find all isomorphisms of the graph G onto itself (i.e., automorphisms of the graph G).

(b) Determine the chromatic number of the graph G and calculate its chromatic polynomial $\chi(G, t)$.

124. Consider the following graph:



(a) Is it possible to draw this graph without lifting the pen from the paper, never retracing an edge, and starting and ending at the same vertex?

(b) Find the chromatic number of the given graph.

(c) Show that the graph is not planar.

125. (a) Find two 4-vertex graphs with different chromatic polynomials.

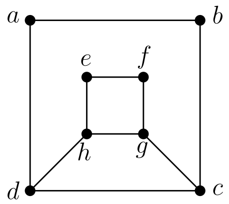
(b) Find two non-isomorphic 4-vertex graphs with the same chromatic polynomial.

(c) Prove that the number of ways to properly color a cycle C_n with $n \geq 3$ vertices using t colors is:

- $(t-1)[(t-1)^{n-1}+1]$ if n is even.
- $(t-1)[(t-1)^{n-1}-1]$ if n is odd.

26 Different math problems

126. Consider the following graph G:



G

(a) Is G an Eulerian graph?

(b) Does G contain an Eulerian trail?

(c) Is G Hamiltonian?

(d) Determine the chromatic number of the graph G.

127. A **wheel graph** W_n for $n \geq 4$ is a graph with n vertices consisting of a cycle C_{n-1} (called the *outer cycle*) and an additional central vertex that is adjacent to every vertex in the cycle.

(a) Draw the wheel graphs W_4, W_5, W_6, and W_7. Start with the outer cycle, then add the central vertex, and connect it to each vertex in the cycle.

(b) For which $n \geq 4$ is W_n an Eulerian graph? For which $n \geq 4$ does W_n have an Eulerian trail?

(c) For which $n \geq 4$ is W_n Hamiltonian?

(d) Show that there exists an orientation of the edges of W_n such that the absolute difference between the in-degree and out-degree of each vertex is at most one.

128. Let W_n be the wheel graph from the previous problem (see Problem 127).

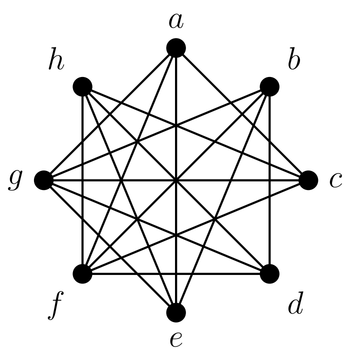
(a) Calculate the clique number $\omega(W_n)$ of the graph W_n (the size of the largest complete subgraph in W_n).

(b) Find the chromatic number $\chi(W_n)$ of the graph W_n.

(c) Show that the chromatic polynomial of W_n is given by:

$$\chi(W_n, t) = t\left((t-2)^{n-1} - (-1)^n(t-2)\right).$$

129. Consider the graph G below and answer the questions. Justify your answers.



G

(a) Is G Hamiltonian?

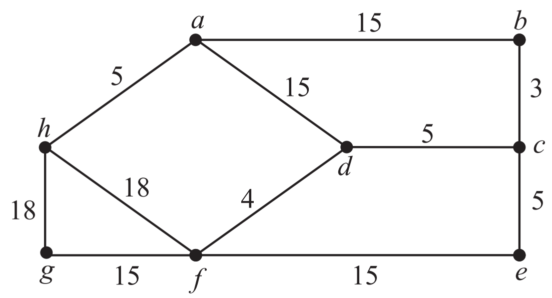
(b) Is G connected?

(c) Is G an Eulerian graph? Does it contain an Eulerian trail?

(d) Is G planar?

(e) Determine $\chi(G)$.

130. Let G be the following weighted graph:



(i) Use Kruskal's greedy algorithm to find a minimum spanning tree of G and specify its total weight. Outline the steps of the algorithm.

(ii) Determine whether the tree found in the previous part is unique, i.e., whether another spanning tree with the same weight can be obtained using Kruskal's method.

(iii) Is G an Eulerian graph? If yes, write the corresponding trail; if no, provide a justification.

(iv) Is G Hamiltonian? Does G have a Hamiltonian path? If yes, specify the cycle/path; if no, justify your answer.

(v) Calculate $\chi(G)$.

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.