Some computational aspects of solvable regular covers of graphs

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Abstract

Given a connected graph X and a group G of its automorphisms we first introduce an approach for constructing all pairwise nonequivalent connected solvable regular coverings $\wp\colon \tilde{X} \to X$ (that is, with a solvable group of covering transformations $\mathrm{CT}(\wp)$) along which G lifts, up to a prescribed order n of $\tilde{X}$.

Next, given a connected solvable regular covering $\wp\colon \tilde{X} \to X$ by means of voltages and a group $G \leq \mathrm{Aut}(X)$ that lifts along $\wp$, we consider algorithms for testing whether the lifted group $\tilde{G}$ is a split extension of $\mathrm{CT}(\wp)$. In computational group theory, methods for testing whether a given extension of permutation groups splits are known. However, in order to apply the existing algorithms, $\tilde{X}$ together with $\mathrm{CT}(\wp)$ and $\tilde{G}$ need to be constructed in the first place, which is far from optimal. Recently, an algorithm avoiding such explicit constructions has been proposed by Malnič and the author (On the Split Structure of Lifted Groups, submitted). We here provide additional details about this algorithm and investigate its performance compared to the one using explicit constructions. To this end, a concrete dataset of solvable regular covers of graphs has been generated by the algorithm mentioned in the first paragraph.

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1 Introduction

Covering graph techniques have proven successful as effective tools to investigate structural properties of mathematical objects. In particular, these techniques enable certain constructions of regular coverings out of graphs we already have in hand. Such constructions have been applied to a number of symmetric graphs with the

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aim to classify particular classes of graphs and maps on surfaces, count the number of graphs in certain families, construct infinite families or to produce catalogues of graphs with prescribed degree of symmetry up to a certain reasonable size. For instance, Djoković [8] used regular coverings to construct the first examples of infinite families of 5-arc-transitive cubic graphs, while a method for constructing certain 5-arc-transitive cubic graphs as regular covers of cubic graphs that are 4- but not 5-arc-transitive was developed by Biggs [1]. Further, the above ideas were applied in constructing the census of semisymmetric cubic graphs on up to 768 vertices [7].

Also, one would like to find algorithms to provide answers to certain natural questions regarding symmetry issues of graphs and their regular coverings. For example, a basic question in this respect is whether a given group $G \leq \mathrm{Aut}(X)$ of automorphisms lifts along a regular covering projection $\wp\colon \tilde{X} \to X$ of connected graphs to a group $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$ (see Preliminaries for the definition). The observation that the covering graph $\tilde{X}$ inherits certain symmetries of the base graph X – subject to the condition that G lifts and satisfies certain structural properties according to its action on X – exhibits the essence of lifting groups of automorphisms. Moreover, the existence of $\tilde{G}$ implies that $\tilde{G}$ is an extension of the group of covering transformations $\mathrm{CT}(\wp)$ by G. It is therefore reasonable to ask, before all else, whether the extension splits. This gives further advantage to reveal additional symmetry properties of $\tilde{X}$. In this context, some algorithmic questions were addressed in [16, 22, 24].

The fact that the structure of $\tilde{X}$ can be conveniently encoded on X in terms of voltages has led to a purely combinatorial description of graph coverings, which, among other things, allows one to study graphs from an algorithmic and computational point of view. The theory of combinatorial graph coverings was developed by Gross and Tucker [11] in the seventies. Malnič, Nedela and Škoviera extended these ideas to a systematic combinatorial treatment of lifting automorphisms along covering projections [15, 17]. Along these lines, a method for constructing admissible cyclic regular covers of connected graphs was proposed by Širáň [25], while elementary abelian regular covers were considered by Malnič, Marušič and Potočnik in [19]. A similar approach was proposed by Du, Kwak and Xu [9] in order to find cyclic or elementary abelian regular coverings of connected graphs. Most recently Conder and Ma employed methods from representation theory in order to classify certain arc-transitive cubic abelian regular covers [5].

The purpose of this paper is twofold. Firstly, given a connected graph X and a group G of its automorphisms, we introduce an approach for constructing connected solvable regular coverings $\wp\colon \tilde{X} \to X$ along which G lifts, up to a prescribed order n of the respective graphs $\tilde{X}$. The idea behind our approach is to exploit methods for finding elementary abelian regular covers. And secondly, given a connected solvable regular covering $\wp\colon \tilde{X} \to X$ combinatorially by means of voltages and a group $G \leq \mathrm{Aut}(X)$ that lifts along $\wp$, we consider algorithms for testing whether $\tilde{G}$ is split extension of $\mathrm{CT}(\wp)$. It is well known that efficient methods for testing whether an extension of permutation groups splits already exist, see for instance [4] and [12, Chapters 7 and 8]. In the above combinatorial setting of graph covers, however, $\tilde{X}$ together with $\mathrm{CT}(\wp)$ and $\tilde{G}$ have to be explicitly constructed – as permutation groups acting on $\tilde{X}$ – before the existing algorithms can be applied. Since these constructions are time and space consuming, the obvious naive algorithm seems to be inappropriate. In [22], an algorithm that avoids such explicit constructions has been briefly described. We continue this line of research: we provide additional details about this algorithm and explore its behaviour in order to demonstrate its superior performance in comparison with the naive one.

The rest of the paper is organized as follows. In Section 2 we give some background material on regular covering projections of graphs and lifting automorphisms, including some details that we require later on. In Section 3 we devise an approach for generating connected solvable regular graph covers. A detailed description of the algorithm from [22] is presented in Section 4. In Section 5 we describe the concrete dataset that we have generated in order to exhibit the capabilities of the proposed algorithm from [22], along with the evaluation results.

2 Preliminaries

2.1 Regular covers of graphs

A *graph* is an ordered quadruple $X = (D, V; \mathrm{beg}, ^{-1})$, where $D(X) = D$ and $V(X) = V$ are disjoint sets of *darts* and *vertices*, respectively, beg is the function assigning to each dart its *initial vertex*, and $^{-1}$ is an arbitrary involution on D that creates edges arising as orbits of $^{-1}$. For a dart x, its *terminal vertex* is the vertex $\mathrm{end}(x) = \mathrm{beg}(x^{-1})$. An edge $e = \{x, x^{-1}\}$ is called a *link* whenever $\mathrm{beg}(x) \neq \mathrm{end}(x)$. If $\mathrm{beg}(x) = \mathrm{end}(x)$, then the respective edge is either a *loop* or a *semi-edge*, depending on whether $x \neq x^{-1}$ or $x = x^{-1}$, respectively.

A *graph homomorphism* $f\colon Y \to X$ is an adjacency preserving mapping taking darts to darts and vertices to vertices, or more precisely, $f(\mathrm{beg}(x)) = \mathrm{beg}(f(x))$ and $f(x^{-1}) = f(x)^{-1}$. An *isomorphism* is a bijective homomorphism. An isomorphism of a graph onto itself is an *automorphism*. All automorphisms of a graph X together with composition of automorphisms constitute the *automorphism group* $\mathrm{Aut}(X)$.

A surjective homomorphism $\wp : \tilde{X} \to X$ is called a *regular covering projection* (or a *regular cover*) if there exists a semiregular subgroup $C \leq \mathrm{Aut}(\tilde{X})$ such that its orbits on vertices and on darts coincide with *vertex fibres* $\wp^{-1}(v)$, $v \in V(X)$, and *dart fibres* $\wp^{-1}(x)$, $x \in D(X)$, respectively. Two regular covering projections $\wp\colon \tilde{X} \to X$ and $\wp'\colon \tilde{X}' \to X$ are *isomorphic* if there exists an automorphism $g \in \mathrm{Aut}(X)$ and an isomorphism $\tilde{g}\colon \tilde{X} \to \tilde{X}'$ such that the following diagram

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\tilde{g}} & \tilde{X}' \\ \wp \downarrow & & \downarrow \wp' \\ X & \xrightarrow{g} & X \end{array}$$

is commutative. If in the above diagram one can choose $g = \mathrm{id}$, then the projections are *equivalent*.

Regular covering projections can be grasped combinatorially as follows. Let X be a graph and let S be an (abstract) group, called the *voltage group*. Assign to each dart x in X a *voltage* $\xi(x) \in S$ such that $\xi(x^{-1}) = (\xi(x))^{-1}$. Further, let

$\mathrm{Cov}(\xi; S)$ be the *derived graph* with vertex-set $V \times S$ and dart set $D \times S$, where $\mathrm{beg}(x, s) = (\mathrm{beg}(x), s)$ and $(x, s)^{-1} = (x^{-1}, s\,\xi(x))$. Then the projection onto the first coordinate is a *derived regular covering projection* $\wp\colon \mathrm{Cov}(\xi; S) \to X$, where the required semiregular subgroup C of $\mathrm{Aut}(\mathrm{Cov}(\xi; S))$ arises from the action of S via left multiplication on itself. Conversely, it can be shown that with any regular covering projection $\wp : \tilde{X} \to X$ we can associate a voltage assignment ξ on X such that the projection derived from ξ is equivalent to $\wp$. Two assignments ξ and ξ' on X are *equivalent* whenever the respective derived regular covering projections are equivalent. For an extensive treatment of graph coverings we refer the reader to [11, 15, 17].

Let $\xi\colon X \to S$ be a voltage assignment and $\wp\colon \mathrm{Cov}(\xi; S) \to X$ the derived regular covering projection of connected graphs. Suppose that the group S has a normal subgroup N. We explain how to combinatorially reconstruct a decomposition of $\wp\colon \mathrm{Cov}(\xi; S) \to X$ arising, up to equivalence, from N. If $q\colon S \to S/N$ is the natural quotient projection, then

$$\wp_q\colon \mathrm{Cov}(\xi_q; S/N) \to X$$

is a regular covering projection, where the respective voltage assignment is given by $\xi_q = q \circ \xi\colon X \to S/N$, see [19]. Moreover, there exists a regular covering projection

$$\bar{\wp}_q\colon \mathrm{Cov}(\bar{\xi}_q; N) \to \mathrm{Cov}(\xi_q; S/N)$$

derived from a voltage assignment $\bar{\xi}_q\colon \mathrm{Cov}(\xi_q; S/N) \to N$, such that the projection $\wp$ is equivalent to the composition $\bar{\wp}_q \circ \wp_q$. In other words, there exists an isomorphism $\alpha\colon \mathrm{Cov}(\xi; S) \to \mathrm{Cov}(\bar{\xi}_q; N)$ such that the diagram

$$\begin{array}{ccc} \mathrm{Cov}(\xi; S) & \xrightarrow{\ \alpha\ } & \mathrm{Cov}(\bar{\xi}_q; N) \\ {\scriptstyle \wp}\downarrow & & \downarrow{\scriptstyle \bar{\wp}_q} \\ X & \xleftarrow{\ \wp_q\ } & \mathrm{Cov}(\xi_q; S/N) \end{array}$$

is commutative. We describe a general recipe for constructing a voltage assignment $\bar{\xi}_q$, to be repeatedly used in Section 4.

Let T be a complete set of representatives of the right cosets of N in S. First note that each $a \in S$ can be uniquely written as $a = nt$, for some $n \in N$ and $t \in T$. For each $t \in T$ and $x \in D(X)$ choose the representative $s_{t,\xi(x)} \in T$ of the right coset $Nt\xi(x)$, that is, $Nt\xi(x) = Ns_{t,\xi(x)}$.

Theorem 2.1. *With the above notation, the assignment* $\bar{\xi}_q\colon \mathrm{Cov}(\xi_q; S/N) \to N$ *is given by*

$$\bar{\xi}_q((x, Nt)) = t\,\xi(x)\,s_{t,\xi(x)}^{-1},$$

where (x, Nt) *is an arbitrary dart in* $\mathrm{Cov}(\xi_q; S/N)$.

Proof. Let (x, Nt) be an arbitrary dart in the derived graph $\mathrm{Cov}(\xi_q; S/N)$, where $x \in D(X)$ and $t \in T$. Then its inverse dart is

$$(x, Nt)^{-1} = (x^{-1}, Nt\xi_q(x)) = (x^{-1}, NtN\xi(x)) = (x^{-1}, Nt\xi(x)) = (x^{-1}, Ns_{t,\xi(x)}).$$

Suppose now that $\bar{\xi}_q((x, Nt)) = n$, for some $n \in N$. Consider the dart $((x, Nt), n')$ in $\mathrm{Cov}(\bar{\xi}_q; N)$, where $n' \in N$ is an arbitrary voltage. Then $((x, Nt), n')$ can be uniquely identified via the isomorphism α with the dart $(x, n't)$ in $\mathrm{Cov}(\xi; S)$. Furthermore, the inverse dart of $((x, Nt), n')$ is

$$((x, Nt), n')^{-1} = ((x, Nt)^{-1}, n' \,\bar{\xi}_q((x, Nt))) = ((x^{-1}, Ns_{t,\xi(x)}), n'n),$$

while $(x, n't)^{-1} = (x^{-1}, n't\xi(x))$ is the inverse dart of $(x, n't)$. Identifying the inverse darts $((x^{-1}, Ns_{t,\xi(x)}), n'n)$ and $(x^{-1}, n't\xi(x))$ via α, we have $n'ns_{t,\xi(x)} = n't\xi(x)$. It follows that

$$n = t\xi(x)s^{-1}_{t,\xi(x)}.$$

The latter expression does not depend on the choice of $n' \in N$, so we can define $\bar{\xi}_q((x, Nt)) = t\xi(x)s^{-1}_{t,\xi(x)}$, as stated. □

2.2 Lifting automorphisms

An automorphism $g \in \mathrm{Aut}(X)$ *lifts along* a regular covering projection $\wp\colon \tilde{X} \to X$ if there exists an automorphism $\tilde{g} \in \mathrm{Aut}(\tilde{X})$ such that the diagram

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\ \tilde{g}\ } & \tilde{X} \\ \wp\downarrow & & \downarrow\wp \\ X & \xrightarrow{\ g\ } & X \end{array}$$

is commutative. The automorphism $\tilde{g}$ then *projects* to g along $\wp$. A group $G \leq \mathrm{Aut}(X)$ *lifts* if all $g \in G$ lift. We call such a covering projection *G-admissible* or *admissible* for short. The collection of all lifts of all elements in G form a subgroup $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$, the *lift* of G. We also say that G is a *projection* of $\tilde{G}$ along $\wp$ (although there may exist a proper subgroup of $\tilde{G}$ which projects to G). In particular, the lift of the trivial group is known as the *group of covering transformations* and is denoted by $\mathrm{CT}(\wp)$. If $\mathrm{CT}(\wp)$ is an elementary abelian or a solvable group, the covering projection $\wp$ is called *elementary abelian* or *solvable*, respectively. Observe that $\mathrm{CT}(\wp)$ is a normal subgroup of $\tilde{G}$ and $\tilde{G}/\mathrm{CT}(\wp) \cong G$ – in other words, $\tilde{G}$ is isomorphic to an extension of $\mathrm{CT}(\wp)$ by G. Furthermore, if G lifts along a given projection $\wp$, then it lifts along any covering projection equivalent to $\wp$. This allows us to study lifts of automorphisms combinatorially in terms of voltage assignments.

Consider now a regular covering projection $\wp\colon \tilde{X} \to X$ of connected graphs. Then we say that $\wp$ is *connected*. Further, the semiregular group C from the definition of a regular covering $\wp$ is $C = \mathrm{CT}(\wp)$, and the voltage assignment $\xi\colon X \to S$ that reconstructs the projection is essentially valued in $S \cong \mathrm{CT}(\wp)$ (viewed as an abstract group).

The problem of finding admissible regular covers of a given connected graph X is very difficult in general. On the other hand, there are efficient methods for finding admissible elementary abelian regular covers. We briefly summarize the one introduced in [19], to be used later on.

For a given prime p, the first homology group $H_1(X; \mathbb{Z}_p)$ is generated by the (directed) cycles of X and is isomorphic to the elementary abelian group $\mathbb{Z}_p^r$, where

r is the Betti number of X. The group $H_1(X;\mathbb{Z}_p)$ is usually viewed as r-dimensional vector space over $\mathbb{Z}_p$. Given an automorphism $\alpha \in \mathrm{Aut}(X)$, there is a natural action of α on $H_1(X;\mathbb{Z}_p)$, since α maps a cycle in X to a cycle in X. This action induces a linear transformation $\alpha^{\#}$ of $H_1(X;\mathbb{Z}_p)$. Choose a spanning tree T of X and exactly one dart from each edge $\{x, x^{-1}\}$ that is not contained in T. Then the sequence $x_1, x_2, \ldots, x_r \in D(X)\backslash D(T)$ of all such darts naturally defines an (ordered) basis $\mathcal{B}_T = \{C_1, C_2, \ldots, C_r\}$ of $H_1(X;\mathbb{Z}_p)$, where C_i is the cycle determined by the spanning tree T and the dart x_i. Further, denote the matrix representation of $\alpha^{\#}$ with respect to the basis $\mathcal{B}_T$ by $M_\alpha \in \mathbb{Z}_p^{r,r}$. Hence a subgroup $G \leq \mathrm{Aut}(X)$ induces a subgroup $M_G = \{M_g \,|\, g \in G\} \leq \mathrm{GL}(r,\mathbb{Z}_p)$. By M_G^t we denote the dual group consisting of all transposes of matrices in M_G.

Theorem 2.2. *([19, Proposition 6.3, Corollary 6.5]) With the notation and assumptions above, let $\xi\colon X \to \mathbb{Z}_p^{d,1}$ be a voltage assignment on X which is trivial on T, and let $Z \in \mathbb{Z}_p^{d,r}$ be the matrix with columns*

$$\xi(x_1), \xi(x_2), \ldots, \xi(x_r).$$

If Z has rank d, then the derived graph $\mathrm{Cov}(\xi;\mathbb{Z}_p^{d,1})$ is connected and the following hold:

(i) A group $G \leq \mathrm{Aut}(X)$ lifts along $\wp\colon \mathrm{Cov}(\xi;\mathbb{Z}_p^{d,1}) \to X$ if and only if the columns of Z^t form a basis of a M_G^t-invariant d-dimensional subspace $\mathcal{S}(\xi)$ of $\mathbb{Z}_p^{r,1} \cong H_1(X;\mathbb{Z}_p)$.

(ii) If $\xi'\colon X \to \mathbb{Z}_p^{d,1}$ is another voltage assignment on X satisfying the above conditions, then $\wp'\colon \mathrm{Cov}(\xi';\mathbb{Z}_p^{d,1}) \to X$ is equivalent to $\wp$ if and only if $\mathcal{S}(\xi') = \mathcal{S}(\xi)$.

In view of Theorem 2.2, we can find all pairwise nonequivalent G-admissible connected elementary abelian regular covers of X in terms of voltages as follows. First, for each M_G^t-invariant subspace U of $\mathbb{Z}_p^{r,1}$ we find a basis $\{u_1, u_2, \ldots, u_d\}$. Next, for each basis $\{u_1, u_2, \ldots, u_d\}$ consider a matrix Z with rows $u_1^t, u_2^t, \ldots, u_d^t$, and then define the voltage assignment $\xi\colon X \to \mathbb{Z}_p^{d,1}$, mapping dart x_i to the i-th column of Z, $i = 1, 2, \ldots, r$, and mapping all darts of T to the trivial voltage. Observe that the choice of a spanning tree together with a sequence $x_1, x_2, \ldots, x_r$ as well as choosing a basis for an invariant subspace is irrelevant as long as we consider regular coverings up to equivalence. Thus, the problem of finding admissible connected elementary abelian regular covers translates to a purely algebraic question of finding invariant subspaces of finite linear groups.

3 Constructing solvable regular covers of graphs

As mentioned earlier, methods for finding connected elementary abelian regular covers of a given connected graph along which a given group of automorphisms lifts are known. We now describe an approach for generating such admissible covers with the group of covering transformations being solvable up to a prescribed order n of the respective covering graphs. The resulting graph covers are explicitly described in terms of voltage assignments. Our approach towards this aim is based on the following observations.

Let $\wp\colon \mathrm{Cov}(\xi;S) \to X$ be a regular covering projection of finite connected graphs derived from a voltage assignment $\xi\colon X \to S$, where S is a solvable group. Further, let $G \le \mathrm{Aut}(X)$ be a group of automorphisms that lifts along the projection $\wp$. Since S is solvable, there exists a series of characteristic subgroups $S = K_0 \rhd K_1 \rhd \cdots \rhd K_n = 1$ with elementary abelian factors K_{j-1}/K_j. In light of the discussion at the end of Subsection 2.1, such a series gives rise to the decomposition

$$\mathrm{Cov}(\xi;S) \cong X_n \overset{\wp_n}{\to} X_{n-1} \to \ldots \to X_1 \overset{\wp_1}{\to} X_0 = X$$

of the covering projection $\wp$ into a series of elementary abelian regular covering projections $\wp_j\colon X_j \to X_{j-1}$ derived from voltage assignments $\xi_j\colon X_{j-1} \to K_{j-1}/K_j$, $1 \le i \le n$. In view of the fact that efficient methods for finding admissible elementary abelian regular covers are known, we would like to turn the story around and find admissible solvable regular covers as compositions of elementary abelian regular covers. This can always be achieved as the following lemma shows.

Lemma 3.1. *Let $\wp\colon \tilde{X} \to X$ be a G-admissible solvable regular cover of connected graphs, and let $\tilde{G}$ be the lift of $G \le \mathrm{Aut}(X)$ along $\wp$. Further, let $\wp'\colon \tilde{X}' \to \tilde{X}$ be a $\tilde{G}$-admissible elementary abelian regular cover of connected graphs. Then the composition $\wp \circ \wp'\colon \tilde{X}' \to X$ is a G-admissible solvable regular cover of connected graphs.*

Proof. First, since $\wp'$ is $\tilde{G}$-admissible, it is also $\mathrm{CT}(\wp)$-admissible. By a result in [26] (c.f. also [27]) that the composition $\wp \circ \wp'$ is regular if and only if $\wp'$ is $\mathrm{CT}(\wp)$-admissible, it follows that $\wp \circ \wp'$ is indeed regular. Second, since the group $\mathrm{CT}(\wp \circ \wp')$ is equal to the lift of $\mathrm{CT}(\wp)$ along $\wp'$, it is isomorphic to an extension of an elementary abelian group $\mathrm{CT}(\wp')$ by a solvable group $\mathrm{CT}(\wp)$. Hence, by the definition of solvability, $\mathrm{CT}(\wp \circ \wp')$ is solvable. Finally, the fact that the lift of $\tilde{G}$ along $\wp'$ projects to G along $\wp \circ \wp'$ implies that $\wp \circ \wp'$ is also G-admissible. This completes the proof. $\square$

In what follows, let X be a finite connected graph and let G be a group of its automorphisms. We start by constructing the set $\mathcal{S}_1$ of all voltage assignments $\xi^{(1)}\colon X \to E$ giving rise to pairwise nonequivalent G-admissible connected elementary abelian regular covers $\wp^{(1)}\colon \mathrm{Cov}(\xi^{(1)};E) \to X$, up to order n. To achieve this, we can use one of the known methods mentioned above. Then for each assignment $\xi^{(1)}$ in $\mathcal{S}_1$ we do the following. First we explicitly construct the derived covering graph $\mathrm{Cov}(\xi^{(1)};E)$ together with the lifted group $\tilde{G}$. With $\mathrm{Cov}(\xi^{(1)};E)$ and $\tilde{G}$ in hand we find all voltage assignments $\bar{\xi}^{(1)}\colon \mathrm{Cov}(\xi^{(1)};E) \to \bar{E}$ giving rise to pairwise nonequivalent $\tilde{G}$-admissible connected elementary abelian regular covers $\bar{\wp}^{(1)}\colon \mathrm{Cov}(\bar{\xi}^{(1)};\bar{E}) \to \mathrm{Cov}(\xi^{(1)};E)$, up to order $\lfloor n/|E| \rfloor$. Then we reconstruct each composition $\wp^{(1)} \circ \bar{\wp}^{(1)}$ in terms of an appropriate voltage assignment. To be specific, we construct $\xi^{(2)}\colon X \to S$ associated with the projection $\wp^{(2)}\colon \mathrm{Cov}(\xi^{(2)};S) \to X$ such that the diagram

$$\begin{array}{ccc}
\mathrm{Cov}(\xi^{(2)};S) & \xrightarrow{\ \alpha\ } & \mathrm{Cov}(\bar{\xi}^{(1)};\bar{E}) \\
\Big\downarrow{\scriptstyle \wp^{(2)}} & & \Big\downarrow{\scriptstyle \bar{\wp}^{(1)}} \\
X & \xleftarrow{\ \wp^{(1)}\ } & \mathrm{Cov}(\xi^{(1)};E)
\end{array}$$

commutes for some isomorphism $\alpha\colon \mathrm{Cov}(\xi^{(2)}; S) \to \mathrm{Cov}(\bar{\xi}^{(1)}; \bar{E})$. Note that $\xi^{(2)}$ might be equivalent to some assignment already constructed either on the present step or on the previous step. If $\xi^{(2)}$ is not equivalent to any of the other assignments, then we add it to the set $\mathcal{S}_2$.

We proceed recursively by applying the same strategy to the set $\mathcal{S}_2$. Since the order of the covering graphs is bounded, we have $\mathcal{S}_{N+1} = \emptyset$ for some N, meaning that no new covers are found after N-steps. The process eventually terminates with the set

$$\mathcal{S}_1 \cup \mathcal{S}_2 \cup \cdots \cup \mathcal{S}_N$$

of voltage assignments associated with pairwise nonequivalent G-admissible connected solvable regular covers of X, up to order n. The code for generating admissible solvable regular covers is given in Algorithm 1.

Input: a finite connected graph X, a group $G \leq \mathrm{Aut}(X)$, a natural number n
Output: a list $\mathcal{C}$ of all voltage assignments on X giving rise to pairwise nonequivalent G-admissible connected solvable regular covers up to order n

find a list $\mathcal{C}$ of all voltage assignments on X giving rise to pairwise nonequivalent G-admissible connected elementary abelian regular covers up to order n;
$i \leftarrow 1$;
while $i \leq |\mathcal{C}|$ **do**
  $(\xi\colon X \to S) \leftarrow \mathcal{C}[i]$;
  construct the graph $\mathrm{Cov}(\xi; S)$ together with the group $\tilde{G}$ along the projection $\wp_\xi$;
  find a list $\mathcal{L}$ of all voltage assignments on $\mathrm{Cov}(\xi; S)$ giving rise to pairwise nonequivalent $\tilde{G}$-admissible connected elementary abelian regular covers up to $\lfloor n/|S| \rfloor$;
  for $\bar{\xi} \in \mathcal{L}$ **do**
    find an assignment ξ' that reconstructs the composition $\wp_\xi \circ \wp_{\bar{\xi}}$;
    if ξ' is not equivalent to any of assignments in $\mathcal{C}$ **then**
      append ξ' to $\mathcal{C}$;
    end if
  end for
  $i \leftarrow i + 1$;
end while
return $\mathcal{C}$

Theorem 3.2. *Given a natural number n, a finite connected graph X and a group G of its automorphisms Algorithm 1 generates all voltage assignments on X giving rise to pairwise nonequivalent G-admissible connected solvable regular covers up to order n.*

Proof. It is clear that at the end of the algorithm covers are pairwise nonequivalent, however, it remains to prove that all representatives of equivalence classes are constructed. Let $\wp\colon \tilde{X} \to X$ be a regular cover of connected graphs, and let

$\wp'\colon \tilde{X}' \to \tilde{X}$ and $\wp''\colon \tilde{X}'' \to \tilde{X}$ be equivalent regular covers of connected graphs. Suppose that compositions $\wp \circ \wp'$ and $\wp \circ \wp''$ are also regular. Since $\wp'$ and $\wp''$ are equivalent, there exists an isomorphism $\tilde{g}\colon \tilde{X}' \to \tilde{X}''$ such that $\wp' = \wp'' \circ \tilde{g}$. Consequently, we have $\wp \circ \wp' = (\wp \circ \wp'') \circ \tilde{g}$, meaning that compositions $\wp \circ \wp'$ and $\wp \circ \wp''$ are equivalent. It is therefore enough to take representatives of equivalence classes in lines 1 and 6. Further, observe that for given isomorphism $\tilde{g}\colon \tilde{X}' \to \tilde{X}''$ and regular cover $r\colon Y \to \tilde{X}'$ of connected graphs, there exist a regular cover $r'\colon Y' \to \tilde{X}''$ of connected graphs and an isomorphism $\tilde{g}'\colon Y \to Y'$ such that $\alpha \circ r = r' \circ \tilde{g}'$. Hence it is enough to consider only nonequivalent covers in line 9, and the result follows from Lemma 3.1 and its preceding discussion. □

4 Testing split extensions

Let $\wp\colon \mathrm{Cov}(\xi; S) \to X$ be a regular covering projection of finite connected graphs derived from a voltage assignment $\xi\colon X \to S$, where S is a solvable group. Further, let $G \leq \mathrm{Aut}(X)$ be a group of automorphisms that lifts along the projection $\wp$. We describe in detail the main features of the algorithm, given in [22], for testing whether G lifts as a split extension of $\mathrm{CT}(\wp)$ by G.

The idea is to decompose a given covering projection $\wp$ into a series of elementary abelian regular covering projections

$$\mathrm{Cov}(\xi; S) \cong X_n \overset{\wp_n}{\to} X_{n-1} \to \ldots \to X_1 \overset{\wp_1}{\to} X_0 = X, \tag{1}$$

arising from a series of characteristic subgroups $S = K_0 \rhd K_1 \rhd \cdots \rhd K_n = 1$ with elementary abelian factors K_{j-1}/K_j. To be specific, each projection $\wp_j\colon X_j \to X_{j-1}$ is derived from a voltage assignment $\xi_j\colon X_{j-1} \to K_{j-1}/K_j$, $1 \leq i \leq n$. In what follows, testing whether $\wp$ is split-admissible for G is done inductively: starting from the bottom up we test whether each intermediate $\wp_i$ is split-admissible for some appropriate group (see below). We now provide technical details (that are omitted in [22]) related to combinatorial reconstruction of all intermediate covering projections.

Initial step. To start with, construct the above series of characteristic subgroups. The method is known, see for instance [12, Chapter 8]. Further, construct a voltage assignment $\xi_1\colon X \to S/K_1$, where $\xi_1 = q_1 \circ \xi$ and $q_1\colon S \to S/K_1$ is the natural quotient projection. Then apply the elementary abelian version for testing whether $\wp_1$ is *G-split-admissible* – that is to say, whether there exists a complement of $\mathrm{CT}(\wp_1)$ within the lift G_1 of G along the projection $\wp_1$, see [22]. If the test is positive, construct the derived covering graph $X_1 = \mathrm{Cov}(\xi_1; S/K_1)$ together with representatives of the conjugacy classes of complements within G_1, then construct a voltage assignment $\bar{\xi}_1\colon X_1 \to K_1$ as described in Theorem 2.1, and continue inductively down the characteristic series.

Inductive step. Let G_j denote the lift of G along the projection $\wp_1 \circ \wp_2 \circ \cdots \circ \wp_j$, and let M_j denote its group of covering transformations $\mathrm{CT}(\wp_1 \circ \wp_2 \circ \cdots \circ \wp_j)$. Assume that at the j-th step we have determined the derived graph X_j together with a voltage assignment $\bar{\xi}_j\colon X_j \to K_j$, and a set $\{U_{ij} \mid 1 \leq i \leq k_j\}$ of representatives of conjugacy classes of complements of M_j in G_j. At the next step, first construct a voltage assignment

$$\xi_{j+1}\colon X_j \to K_j/K_{j+1},$$

where $\xi_{j+1} = q_{j+1} \circ \bar{\xi}_j$ and $q_{j+1}\colon K_j \to K_j/K_{j+1}$. For each U_{ij}, $1 \le i \le k_j$, then do the following:

(a) test whether $\wp_{j+1}$ is U_{ij}-split-admissible, in other words, check whether there exists a complement of $\mathrm{CT}(\wp_{j+1})$ in $\tilde{U}_{ij}$, where $\tilde{U}_{ij}$ is the lift of U_{ij} along $\wp_{j+1}$;

(b) if a complement exists, construct the graph $X_{j+1} = \mathrm{Cov}(\xi_{j+1}; K_j/K_{j+1})$ (only if it has not yet been constructed for some previous index i) together with the set $\mathcal{K}_{ij}$ of complements of $\mathrm{CT}(\wp_{j+1})$ in $\tilde{U}_{ij}$, and then find a set $\mathcal{C}_{ij} = \{C_{ijk} \,|\, 1 \le k \le s_{ij}\}$ of representatives of the orbits of $\tilde{N}_{M_j}(U_{ij})$ acting on $\mathcal{K}_{ij}$ by conjugation, where $\tilde{N}_{M_j}(U_{ij})$ is the lift of the normalizer $N_{M_j}(U_{ij})$ along $\wp_{j+1}$ (see Figure 1).

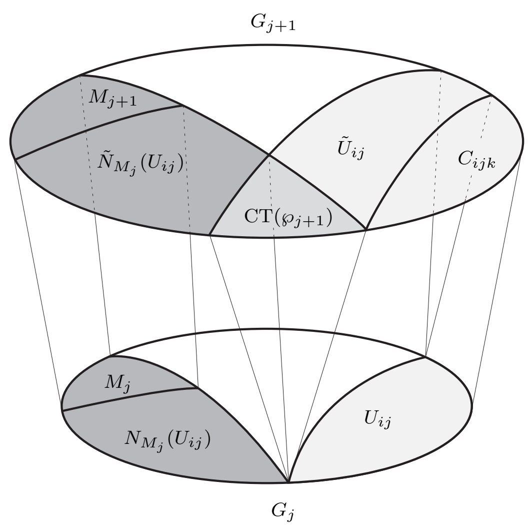

Figure 1: Lifting of complements.

If for all indices $i = 1, 2, \dots, k_j$ the test in (a) is negative, then the projection $\wp$ is not G-split-admissible, and the algorithm stops. Otherwise, construct a voltage assignment $\bar{\xi}_{j+1}\colon X_{j+1} \to K_{j+1}$, and continue with the process. Note that at the $(n-1)$-th step, as soon as the test in (a) is positive for some index i, the projection $\wp$ is G-split-admissible, and the algorithm stops.

Remark 4.1. In order to decide whether there exists a complement as in (a) above, we use the elemetary abelian version of the algorithm given in [22]. The problem reduces to testing solvability of a certain system of linear equations over prime fields. The set $\mathcal{K}_{ij}$ of complements is then in bijective correspondence with all solutions of that system.

Observe that each complement of $\mathrm{CT}(\wp_{j+1})$ in $\tilde{U}_{ij}$ is also a complement of M_{j+1} in G_{j+1}. Moreover, the set $\mathcal{C}_{1j} \cup \mathcal{C}_{2j} \cup \cdots \cup \mathcal{C}_{k_j j}$ is a complete and irredundant set of representatives of conjugacy classes of complements of M_{j+1} in G_{j+1}. For a more detailed discussion about this topic we refer the reader to [4] and [12, Chapter 8].

5 Experiments

In order to evaluate the algorithm described in the previous section as a method for testing whether a given G-admissible solvable regular cover $\wp\colon \mathrm{Cov}(\xi; S) \to X$ is G-split-admissible, we compare its performance with the naive algorithm that consists of the following two steps:

(i) explicitly construct the covering graph $\mathrm{Cov}(\xi; S)$ together with the group of covering transformations $\mathrm{CT}(\wp)$ and the lifted group $\tilde{G}$ of G along $\wp$ – as permutation groups acting on the covering graph, and

(ii) apply the known method for testing whether a given extension of permutation groups splits.

5.1 Test environment

We have implemented both algorithms in the Magma language [2] as commands `IsSplitAdmissible` and `IsSplitAdmissibleNaive`. In this paper, we call them ISA and ISAN for short. A built–in command `HasComplement` has been used in step (ii) of ISAN algorithm. Both algorithms were run on an 2.93 GHz Quad-Core Intel® Xeon® processor X7350 at the Faculty of Mathematics and Physics, University of Ljubljana. The source code of ISA and ISAN is available online at `http://osebje.famnit.upr.si/~ rok.pozar`.

5.2 Dataset

Tools for constructing connected elementary abelian regular graph covers admitting various types of subgroups of automorphism group have been successfully used for a number of small graphs, for instance, the complete graphs K_4 [20] and K_5 [13], the Möbius-Kantor graph $G(8, 3)$ [18], the complete bipartite graph $K_{3,3}$ [10, 20], the Petersen graph $G(5, 2)$ [21], and the Heawood graph [19, 23].

For experimental purposes, however, our aim is to construct a dataset consisting not only of elementary abelian regular covers, but also of solvable ones. Therefore, we have implemented Algorithm 1 in Magma. The reader should be aware that we are omitting the details of the implementation. We only mention here the following. First, recall that finding admissible elementary abelian regular covers is equivalent to finding invariant subspaces of finite linear groups. Further, the latter translates into the problem of finding submodules over a matrix algebra, for which the method is known [14] and is already implemented in Magma. The performance and range of Algorithm 1 depends greatly on finding submodules. Called `SolvableCovers`, our implementation of Algorithm 1 is available online at `http://osebje.famnit.upr.si/~ rok.pozar`.

Running `SolvableCovers`, we prepared a dataset of admissible connected solvable regular covering projections combinatorially in terms of voltages for four small graphs K_5, $K_{3,3}$, $G(5, 2)$ and $G(8, 3)$ as well as for two larger graphs, namely, for the Ljubljana graph $\mathcal{L}$ [6] and the graph F258A from the Foster census [3]. The following theorem summarizes properties of the dataset.

Theorem 5.1. *Up to equivalence of covering projections there are exactly* *(a)* 18 *connected* $\mathrm{Aut}(K_5)$*-admissible solvable regular covers of* K_5 *up to order* 1500*;*

(b) 14 *connected* $\mathrm{Aut}(K_{3,3})$*-admissible solvable regular covers of* $K_{3,3}$ *up to order* 2000*;*

(c) 17 *connected* $\mathrm{Aut}(G(5,2))$*-admissible solvable regular covers of* $G(5,2)$ *up to order* 3000*;*

(d) 45 *connected* $\mathrm{Aut}(G(8,3))$*-admissible solvable regular covers of* $G(8,3)$ *up to order* 1500*;*

(e) 29 *connected* $\mathrm{Aut}(\mathcal{L})$*-admissible regular covers of* $\mathcal{L}$ *up to order* 3000*;*

(f) 18 *connected* $\mathrm{Aut}(F258A)$*-admissible regular covers of* $F258A$ *up to order* 5000*.*

Note that in (e) and (f) of Theorem 5.1 we have actually computed all regular covers, since the orders of voltage groups are smaller than the order of the alternating group Alt(5) – the smallest non-solvable group. Times (CPU times) required for the construction of covers are given in Table 1.

Table 1: Computational times for all solvable covers

Base graph	$t(s)$
K_5	357
$K_{3,3}$	43
$G(5,2)$	242
$G(8,3)$	107
$\mathcal{L}$	304
F258A	622

5.3 Evaluation results

Both algorithms have been compared with respect to execution time (CPU time). In cases where more than one covering graph of the same order exists in the dataset, only one has been chosen among them. Experimental results are gathered in Tables 2-7. The first column shows the order of the covering graph, while the second one describes the three possible types of voltage groups: solvable, but not abelian; abelian, but not elementary abelian; elementary abelian. In addition, the third column identifies the voltage group by its library number in the database of small groups in MAGMA. Execution times given in seconds are displayed in the fourth and the fifth column (for ISA and ISAN, respectively). The last column indicates whether the corresponding covering projection is split-admissible for the full automorphism group. The performance of both algorithms is also shown graphically in Figure 2.

As can be seen from Tables 2-7 and Figure 2, it is clear that ISA algorithm outperforms ISAN algorithm. We point out that this is due to step (i) of ISAN, since the explicit construction of the derived graph $\mathrm{Cov}(\xi; S)$ together with groups $\mathrm{CT}(\wp)$ and $\tilde{G}$ is time consuming. On the other hand, ISA never explicitly constructs neither $X_n \cong \mathrm{Cov}(\xi; S)$ nor $M_n \cong \mathrm{CT}(\wp)$ or $G_n \cong \tilde{G}$, but only constructs derived graphs X_j, $j = 1, 2, \ldots, n-1$, in decomposition (1) from Section 4, if needed. These graphs are usually much smaller.

Table 2: Performance comparison for the Möbius-Kantor graph

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\mathrm{ISA}}(s)$	$t_{\mathrm{ISAN}}(s)$	Split?
48	Elementary abelian	$\langle 3,1\rangle$	0.010	0.010	true
64	Elementary abelian	$\langle 4,2\rangle$	0.010	0.050	false
128	Elementary abelian	$\langle 8,5\rangle$	0.010	0.070	false
144	Elementary abelian	$\langle 9,2\rangle$	0.010	0.070	true
192	Solvable	$\langle 12,3\rangle$	0.040	0.110	true
256	Abelian	$\langle 16,2\rangle$	0.010	0.170	false
384	Solvable	$\langle 24,3\rangle$	0.290	0.440	true
400	Elementary abelian	$\langle 25,2\rangle$	0.000	0.340	true
432	Solvable	$\langle 27,3\rangle$	0.130	0.500	false
512	Solvable	$\langle 32,47\rangle$	0.010	0.830	false
576	Solvable	$\langle 36,11\rangle$	0.120	0.790	true
768	Solvable	$\langle 48,49\rangle$	0.000	1.530	false
784	Elementary abelian	$\langle 49,2\rangle$	0.000	1.170	true
1024	Solvable	$\langle 64,262\rangle$	0.010	3.160	false
1152	Solvable	$\langle 72,25\rangle$	1.550	3.240	true
1200	Solvable	$\langle 75,2\rangle$	0.020	2.350	true
1296	Solvable	$\langle 81,3\rangle$	0.140	3.960	false

Table 3: Performance comparison for the complete graph K_5

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\mathrm{ISA}}(s)$	$t_{\mathrm{ISAN}}(s)$	Split?
10	Elementary abelian	$\langle 2,1\rangle$	0.000	0.000	true
30	Solvable	$\langle 6,1\rangle$	0.010	0.030	true
120	Solvable	$\langle 24,12\rangle$	0.050	0.160	true
160	Elementary abelian	$\langle 32,51\rangle$	0.010	0.210	true
240	Solvable	$\langle 48,28\rangle$	0.520	0.580	false
320	Elementary abelian	$\langle 64,267\rangle$	0.010	0.690	true
480	Solvable	$\langle 96,230\rangle$	0.350	1.760	true
625	Elementary abelian	$\langle 125,5\rangle$	0.000	1.600	false
640	Solvable	$\langle 128,2326\rangle$	1.530	2.690	true
960	Solvable	$\langle 192,1542\rangle$	1.530	6.050	true
1250	Abelian	$\langle 250,15\rangle$	0.020	6.170	false
1280	Solvable	$\langle 256,55642\rangle$	1.670	9.800	true

In the case of elementary abelian voltage groups, ISA even never explicitly constructs any of the derived graphs X_j, and hence it does not depend on the order of the voltage group. Consequently, with the base graph fixed, the execution times are rather constant for ISA, while the execution times for ISAN grow together with the order of the voltage group. Therefore, in the case of elementary abelian voltage groups the difference between the compared algorithms is most pronounced. On the other hand, in the case when voltage groups are not elementary abelian and ISA has to construct all graphs X_j up to $j = n-1$, together with the corresponding voltage assignments and complements – for instance, when the covering projection $\wp$ is G-split-admissible – the difference between execution times is a bit less pronounced.

Table 4: Performance comparison for the complete bipartite graph $K_{3,3}$

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\text{ISA}}(s)$	$t_{\text{ISAN}}(s)$	Split?
18	Elementary abelian	$\langle 3,1 \rangle$	0.000	0.000	false
96	Elementary abelian	$\langle 16,14 \rangle$	0.000	0.060	true
162	Elementary abelian	$\langle 27,5 \rangle$	0.000	0.100	false
192	Solvable	$\langle 32,49 \rangle$	0.070	0.170	false
288	Abelian	$\langle 48,52 \rangle$	0.080	0.260	false
384	Solvable	$\langle 64,239 \rangle$	0.080	0.540	false
486	Abelian	$\langle 81,11 \rangle$	0.000	0.570	false
576	Solvable	$\langle 96,224 \rangle$	0.080	1.070	false
1152	Solvable	$\langle 192,1541 \rangle$	0.010	2.980	false
1458	Abelian	$\langle 243,61 \rangle$	0.010	4.780	false
1536	Solvable	$\langle 256,8935 \rangle$	0.090	7.930	true

Table 5: Performance comparison for the Petersen graph

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\text{ISA}}(s)$	$t_{\text{ISAN}}(s)$	Split?
20	Elementary abelian	$\langle 2,1 \rangle$	0.000	0.010	true
40	Elementaryabelian	$\langle 4,2 \rangle$	0.010	0.010	true
80	Solvable	$\langle 8,4 \rangle$	0.020	0.040	false
360	Solvable	$\langle 36,10 \rangle$	0.020	0.330	true
640	Elementary abelian	$\langle 64,267 \rangle$	0.000	1.400	true
720	Solvable	$\langle 72,24 \rangle$	0.020	1.300	false
1080	Solvable	$\langle 108,17 \rangle$	0.610	2.310	true
1250	Elementary abelian	$\langle 125,5 \rangle$	0.000	3.310	false
1280	Solvable	$\langle 128,2321 \rangle$	1.770	5.560	false
1620	Solvable	$\langle 162,54 \rangle$	0.020	6.850	true
2160	Solvable	$\langle 216,33 \rangle$	0.030	9.390	false
2500	Abelian	$\langle 250,15 \rangle$	0.030	12.680	false
2560	Solvable	$\langle 256,55628 \rangle$	1.810	22.190	false

Table 6: Performance comparison for the Ljubljana graph $\mathcal{L}$

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\text{ISA}}(s)$	$t_{\text{ISAN}}(s)$	Split?
336	Elementary abelian	$\langle 3,1 \rangle$	0.010	0.220	true
448	Elementary abelian	$\langle 4,2 \rangle$	0.010	0.460	true
784	Elementary abelian	$\langle 7,1 \rangle$	0.020	0.940	true
896	Solvable	$\langle 8,4 \rangle$	0.650	1.730	true
1008	Elementary abelian	$\langle 9,2 \rangle$	0.010	1.850	true
1344	Solvable	$\langle 12,3 \rangle$	0.560	3.190	true
1456	Elementary abelian	$\langle 13,1 \rangle$	0.010	3.080	true
1792	Abelian	$\langle 16,2 \rangle$	0.630	5.510	true
2128	Elementary abelian	$\langle 19,1 \rangle$	0.020	6.420	true
2352	Solvable	$\langle 21,1 \rangle$	0.600	9.270	true
2688	Solvable	$\langle 24,11 \rangle$	3.090	14.110	true
2800	Elementary abelian	$\langle 25,2 \rangle$	0.010	13.320	true

Table 7: Performance comparison for the graph F258A

Order of covering graph	Type of voltage group	Library number of voltage group	$t_{\mathrm{ISA}}(s)$	$t_{\mathrm{ISAN}}(s)$	Split?
774	Elementary abelian	$\langle 3, 1\rangle$	0.050	0.870	false
1032	Elementary abelian	$\langle 4, 2\rangle$	0.040	1.880	true
1806	Elementary abelian	$\langle 7, 1\rangle$	0.040	4.380	true
2064	Solvable	$\langle 8, 4\rangle$	2.660	7.990	true
2322	Elementary abelian	$\langle 9, 2\rangle$	0.090	8.240	false
3096	Abelian	$\langle 12, 5\rangle$	2.720	16.750	false
3354	Elementary abelian	$\langle 13, 1\rangle$	0.040	15.230	true
4128	Abelian	$\langle 16, 2\rangle$	2.670	25.470	true
4902	Elementary abelian	$\langle 19, 1\rangle$	0.040	32.310	true

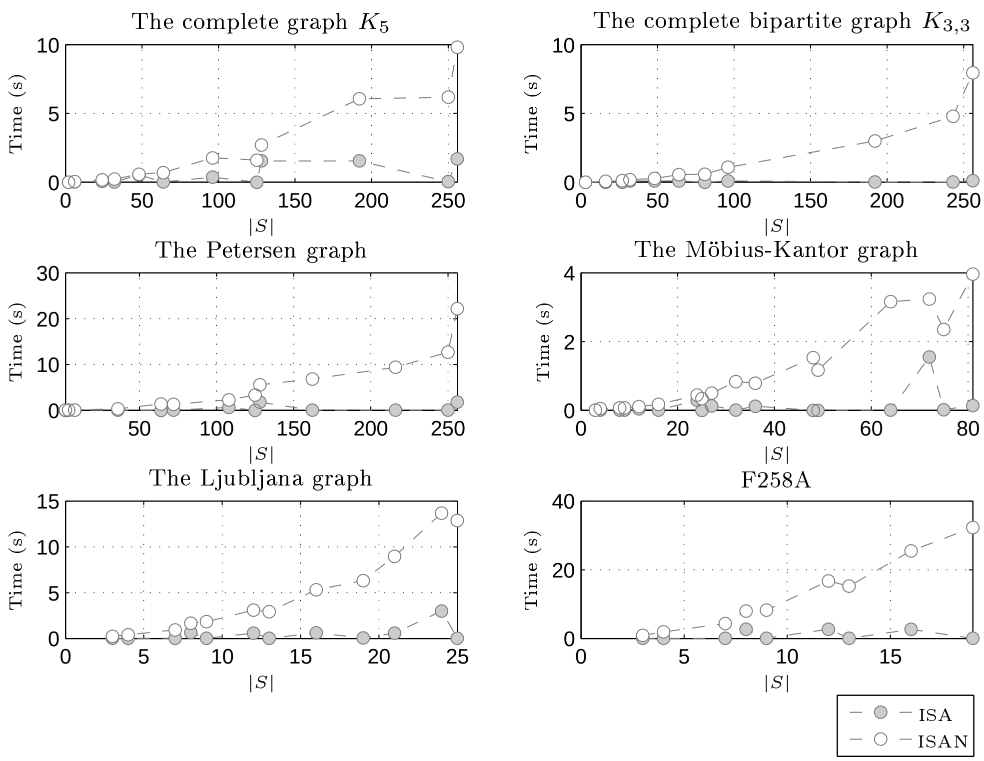

Figure 2: Behavior of the two methods.

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