1 Homework 1 (Spaces and subspaces)

1. Let
$$\mathcal{L} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 | x_1 + x_2 = 0, -x_1 + 2x_2 + x_3 = 0\right\}.$$
Show that $\mathcal{L}$ is subspace of vector space $\mathbb{R}^3$.

2. Let $\mathcal{V} = \mathbb{R}^n$ and let $(a_1, a_2, ..., a_n)^\top$ be some fixed vector from $\mathcal{V}$. Show that the family of all elements $(x_1, x_2, ..., x_n)^\top$ from $\mathcal{V}$ with the property $a_1x_1 + ... + a_nx_n = 0$ is subspace of vector space $\mathcal{V}$. In another words, show that
$$\mathcal{M} = \{(x_1, x_2, ..., x_n)^\top \in \mathcal{V} \,|\, a_1x_1 + ... + a_nx_n = 0\}$$
is subspace of $\mathcal{V}$.

3. Let $\mathcal{V}$ denote vector space of all matrices of form 2×2 over the field of real numbers. Let $\mathcal{W}_1$ be the set of all matrices of form
$$\begin{pmatrix} x & -x \\ y & z \end{pmatrix}$$
and let $\mathcal{W}_2$ be the set of all matrices of the form
$$\begin{pmatrix} a & b \\ -a & c \end{pmatrix}.$$
Show that $\mathcal{W}_1$ and $\mathcal{W}_2$ are subspaces of $\mathcal{V}$.

4. Let
$$\mathcal{V} = \left\{ \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} \in \mathrm{Mat}_{2\times2}(\mathbb{C}) \,\middle|\, z_1 - 2\overline{z_2} + z_3 = 0, z_1 + \overline{z_2 + z_3} + z_4 = 0 \right\}$$
be a given set. Show that $\mathcal{V}$ is real subspace of vector space $\mathrm{Mat}_{2\times2}(\mathbb{C})$.