# New Geometric Representations and Domination Problems on Tolerance and Multitolerance Graphs

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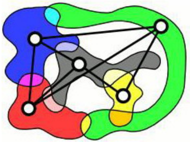
# Intersection graphs

## Definition

An undirected graph $G = (V, E)$ is called an intersection graph, if each vertex $v \in V$ can be assigned to a set $S_v$, such that two vertices of $G$ are adjacent if and only if the corresponding sets have a nonempty intersection, i.e. $E = \{uv \mid S_u \cap S_v \neq \emptyset\}$.



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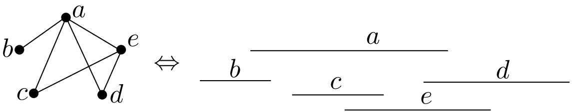
## Definition

A graph $G$ is called an interval graph, if $G$ is the intersection graph of a set of intervals on the real line.



# Tolerance graphs

**Definition (Golumbic, Monma, 1982)**

A graph $G = (V, E)$ is called a tolerance graph, if there is a set $I = \{I_v \mid v \in V\}$ of intervals and a set $t = \{t_v \mid v \in V\}$ of positive numbers, such that $uv \in E$ if and only if $|I_u \cap I_v| \geq \min\{t_u, t_v\}$.

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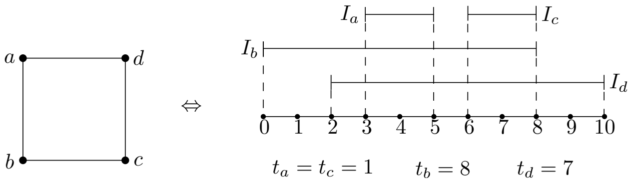

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Otherwise, if $t_v > |I_v|$, $v$ is called an unbounded vertex.

Tolerance graphs have important applications
[Golumbic, Trenk, *Tolerance graphs*, 2004]:

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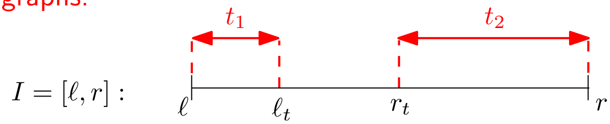
In applications of BLAST, some genomic regions may be:

- biologically less significant, or
- more error prone than others

$\Longrightarrow$ we want to treat several genomic parts non-uniformly.

Multitolerance graphs:



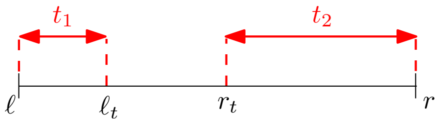

- from left and right: different tolerances.

# Multitolerance graphs

## Motivation and definition

Multitolerance graphs:

$I = [\ell, r]:$




- from left and right: different tolerances.
- in the interior part: tolerate a convex combination of $t_1$ and $t_2$.

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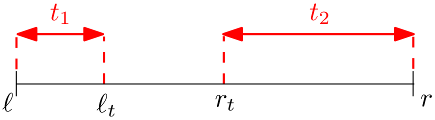


Formally:

- $\mathcal{I}(I, \ell_t, r_t) = \{\lambda \cdot [\ell, \ell_t] + (1-\lambda) \cdot [r_t, r] \ : \ \lambda \in [0,1]\}$
  (convex hull of $[\ell, \ell_t]$ and $[r_t, r]$)

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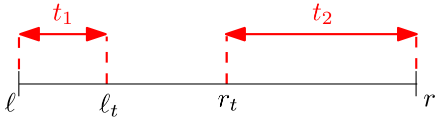


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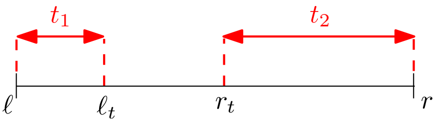


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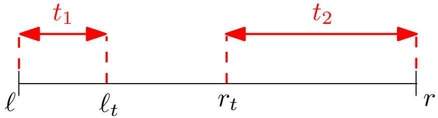


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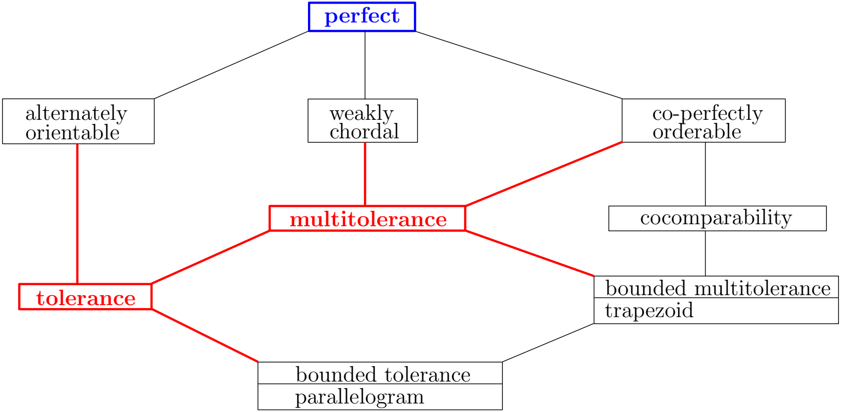
  - either $\tau = \mathcal{I}(I, \ell_t, r_t)$ for two values $\ell_t, r_t \in I$ (bounded case),
  - or $\tau = \mathbb{R}$ (unbounded case).
- In a multitolerance graph $G = (V, E)$, $uv \in E$ whenever:
  - there exists a tolerance-interval $Q_u \in \tau_u$ such that $Q_u \subseteq I_v$, or
  - there exists a tolerance-interval $Q_v \in \tau_v$ such that $Q_v \subseteq I_u$.

# Complete classification in the hierarchy of perfect graphs




[Golumbic, Trenk, *Tolerance Graphs*, 2004]
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- Some (few) algorithms used the (multi)tolerance representation:
  [Parra, *Discr. Appl. Math.*, 1998]
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- It seems to be essential to assume (some) given representation:
  - Tolerance graphs are NP-complete to recognize
    [Mertzios, Sau, Zaks, *STACS*, 2010; *SIAM J. Comp.*, 2011]
  - Recognition of multitolerance graphs: Open !

# Tolerance and multitolerance graphs

## Previously known models

- Succinct intersection models are known for:
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  - general tolerance graphs (3D-parallelepiped representation) [Mertzios, Sau, Zaks, *SIAM J. Discr. Math.*, 2009]

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- These representations enabled the design of algorithms:
  - for clique, coloring, independent set, ...
  - in most cases with (optimal) $O(n \log n)$ running time

# Tolerance and multitolerance graphs

## Previously known models

- In spite of research in the area since [Golumbic, Monma, 1982]:
  - a few problems remained open for (multi)tolerance graphs
  - Dominating Set, Hamiltonian Cycle
    [Spinrad, *Efficient Graph Representations*, 2003]

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  - Dominating Set, Hamiltonian Cycle
    [Spinrad, *Efficient Graph Representations*, 2003]
- both these problems are:
  - NP-complete on weakly chordal graphs
    [Booth, Johnson, *SIAM J. Computing*, 1982]
    [Müller, *Discr. Math*, 1996]
  - polynomial on bounded (multi)tolerance (and cocomparability) graphs
    [Kratsch, Stewart, *SIAM J. Discr. Math*, 1993]
    [Deogun, Steiner, *SIAM J. Computing*, 1994]
- the known models do not provide (enough) insight for these problems

$\Rightarrow$ new models are needed !

# Our results

- New geometric representations:
  - shadow representation for multitolerance graphs
  - special case: horizontal shadow representation for tolerance graphs

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- New geometric representations:
  - shadow representation for multitolerance graphs
  - special case: horizontal shadow representation for tolerance graphs
- Applications of these new models:
  - Dominating Set is APX-hard on multitolerance graphs (i.e. no PTAS unless P = NP)
  - Dominating Set is polynomially solvable on tolerance graphs
  - Independent Dominating Set is polynomially solvable on multitolerance graphs (by a sweep-line algorithm)

# Our results

- New geometric representations:
  - shadow representation for multitolerance graphs
  - special case: horizontal shadow representation for tolerance graphs

- Implications of the new representations:
  - we can reduce optimization problems on these graphs
    $\longrightarrow$ to problems in computational geometry
  - Dominating Set is the first problem with different complexity in tolerance & multitolerance graphs

    $\longrightarrow$ surprising dichotomy result
  - useful for sweep-line algorithms

# Bounded multitolerance graphs

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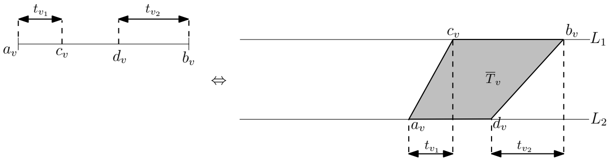

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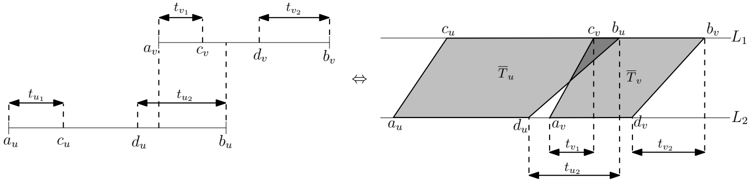

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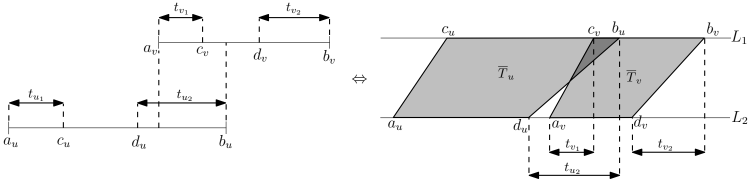


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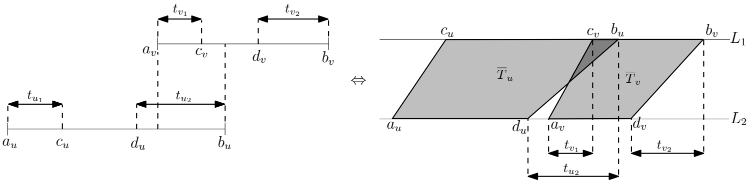


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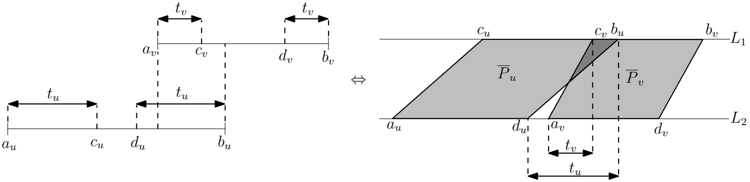
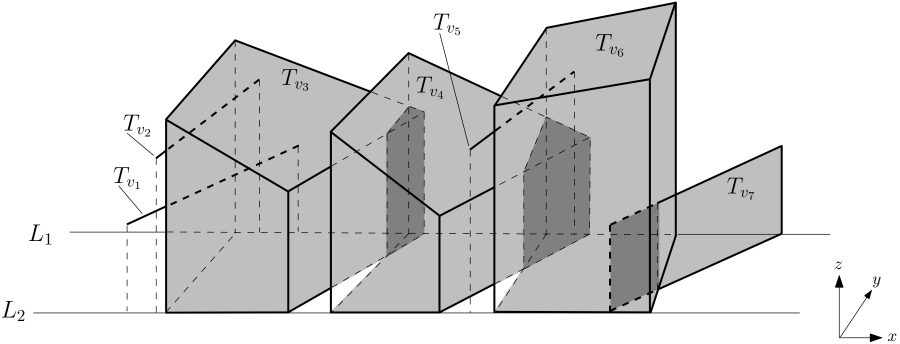


# The trapezoepiped representation

## A 3D-intersection model for multitolerance graphs

- bounded vertices $\longrightarrow$ 3D-trapezoepipeds



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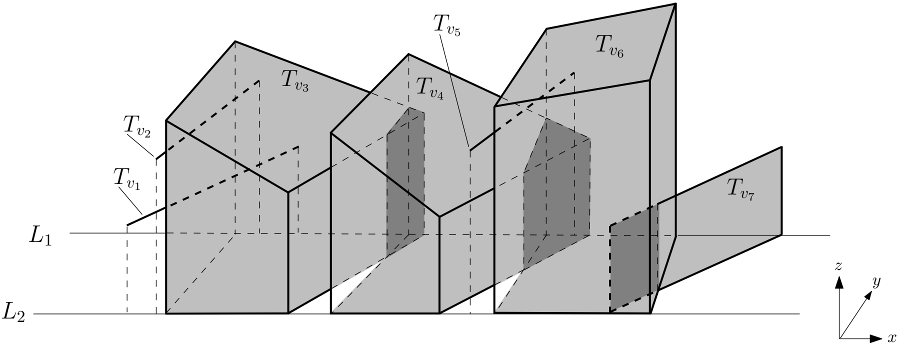

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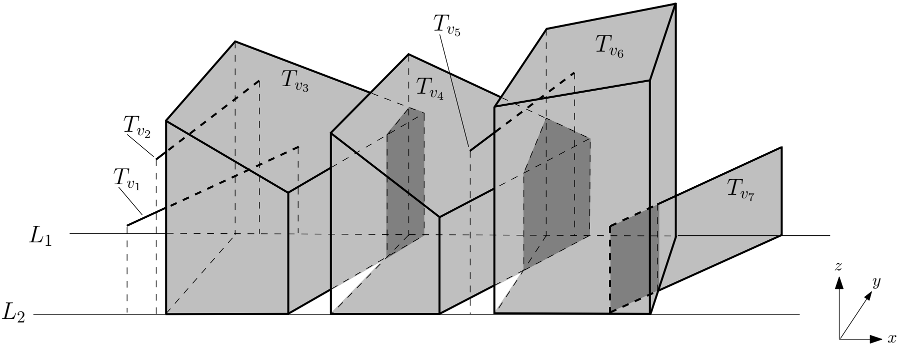
⇒ an intersection model for multitolerance graphs: [Mertzios, *SODA*, 2011; *Algorithmica*, 2014]



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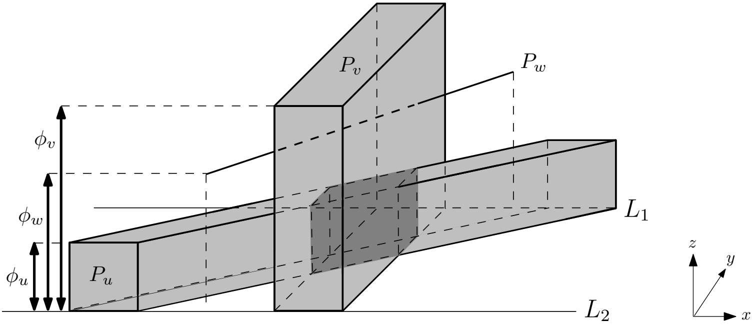
- Special case: parallelepiped representation for tolerance graphs: [Mertzios, Sau, Zaks, *SIAM J. Discr. Math.*, 2009]



# The shadow representation

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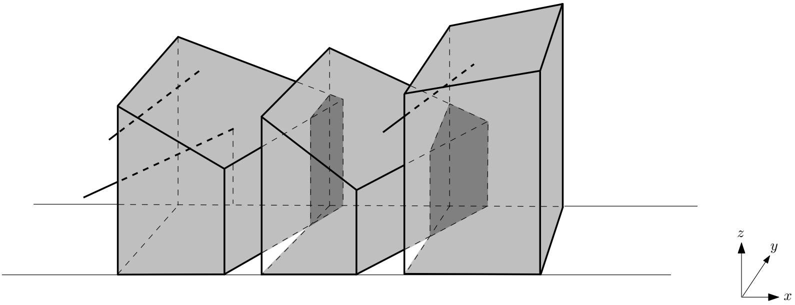

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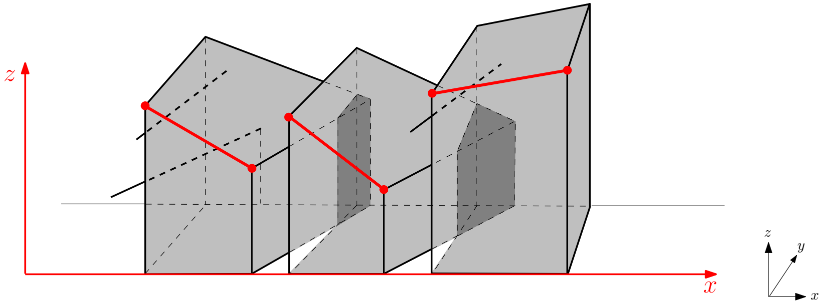

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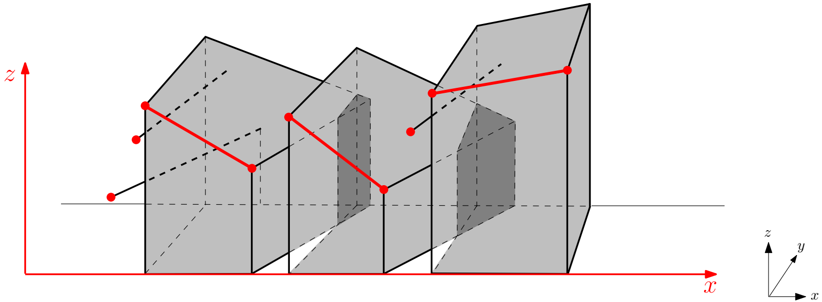

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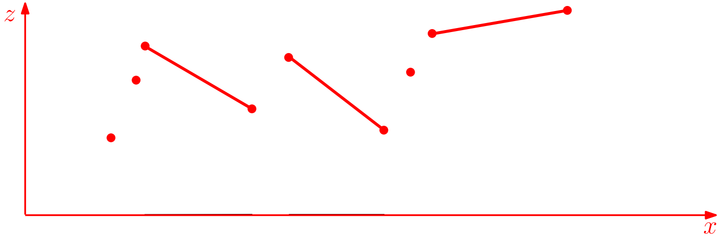

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## Definition

The shadow representation of a multitolerance graph $G$ is a tuple $(\mathcal{P}, \mathcal{L})$:

- $\mathcal{P}$ is the set of all points $p_v$ and
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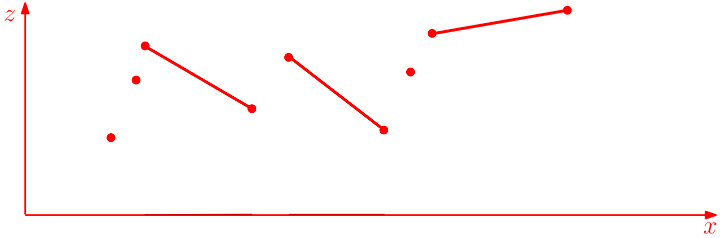

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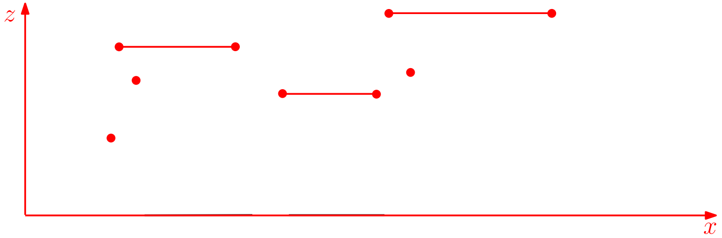
- Special case: tolerance graphs
- parallelepipeds $\Rightarrow$ horizontal line segments

$\Rightarrow$ horizontal shadow representation



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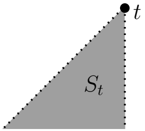
**Answer:** We exploit the "shadows" of the line segments and the points.

# The shadow representation

## Definition (shadow)

- For a point $t = (t_x, t_y) \in \mathbb{R}^2$ the shadow of $t$ is the region $S_t = \{(x, y) \in \mathbb{R}^2 : x \leq t_x,\ y - x \leq t_y - t_x\}$.



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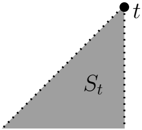
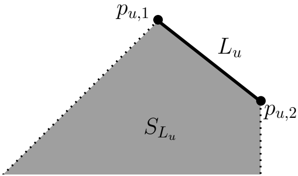
- For every line segment $L_u$ the shadow of $L_u$ is the region $S_{L_u} = \bigcup_{t \in L_u} S_t$.



# The shadow representation

The shadows capture all adjacencies:

**Lemma**

*Let $G = (V, E)$ be a multitolerance graph and $u, v$ be bounded vertices. Then $uv \in E$ if and only if $L_v \cap S_{L_u} \neq \emptyset$ or $L_u \cap S_{L_v} \neq \emptyset$.*

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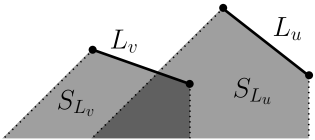

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The shadows capture all adjacencies:

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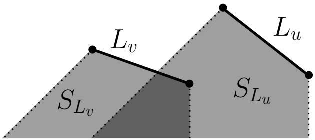


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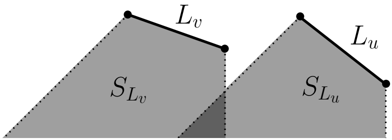

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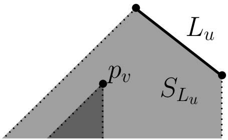

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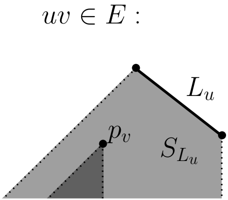


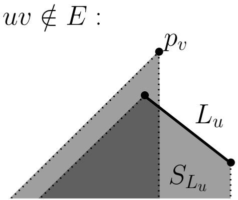

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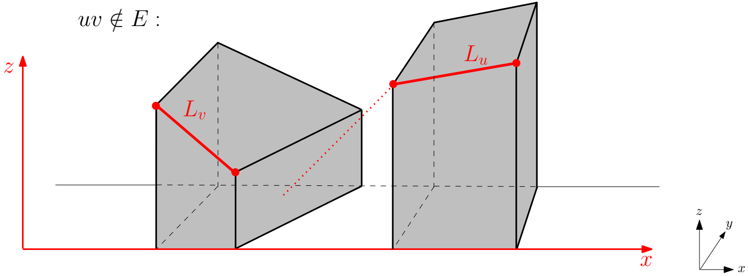
Main idea for the adjacencies:



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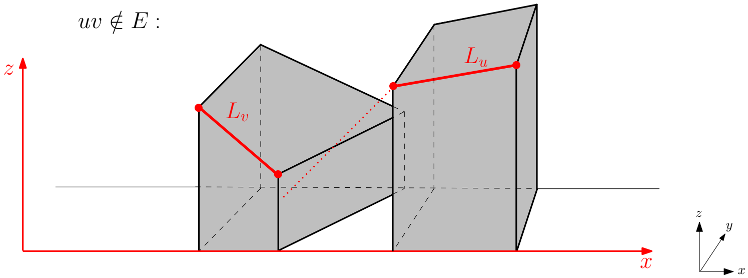

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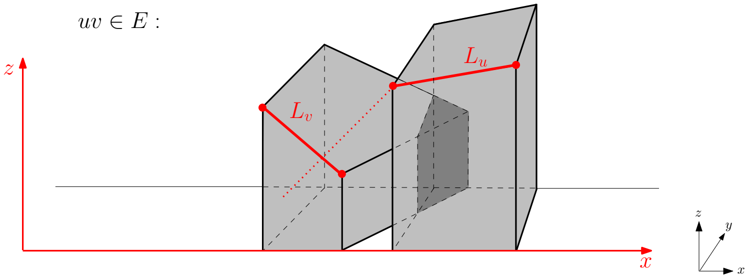

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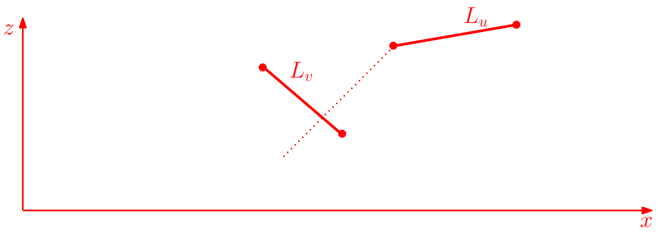


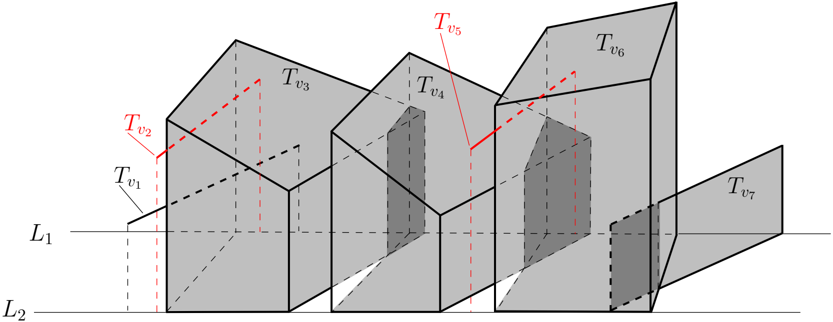

## Observation

*The shadow representation is not an intersection model.*

# Inevitable vertices

## Definition

Let $v$ be an unbounded vertex of a multitolerance graph $G$ (in a certain trapezoepiped representation). If making $v$ a bounded vertex creates a new edge in $G$, then $v$ is called inevitable.



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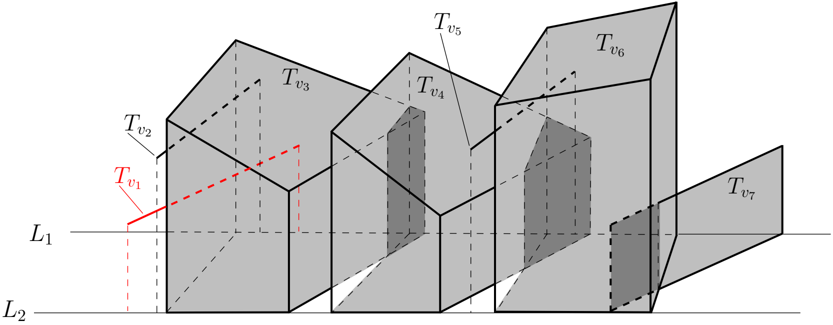
Otherwise, $v$ is called evitable.



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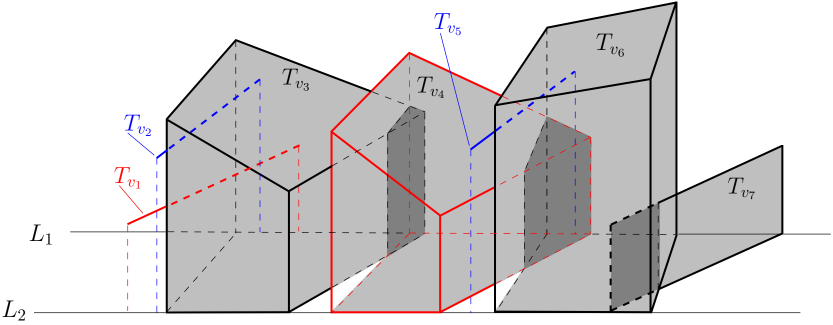
A vertex $u$ is called a hovering vertex of $v$ if $T_v$ lies above $T_u$.



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In a shadow representation:

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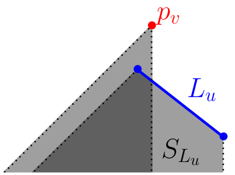

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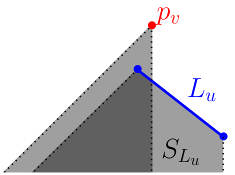


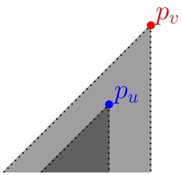

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*Given a trapezoepiped representation of a multitolerance graph $G$, a canonical representation of $G$ can be computed in $O(n \log n)$ time.*

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## Definition

A shadow representation of a multitolerance graph $G$ is called canonical if it can be obtained by a canonical trapezoepiped representation.

In the algorithms:

- it is useful to assume canonical representations

# Dominating set on tolerance graphs

W.l.o.g. we assume:

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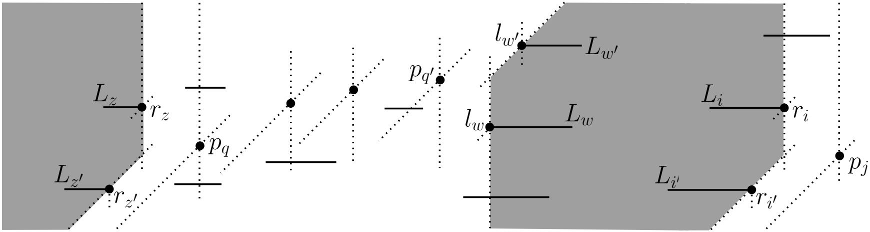
Therefore:

- an unbounded vertex $v$ in the solution "cuts" the representation into "left" and "right"

$\Rightarrow$ dynamic programming, using the position of the unbounded vertices

# Dominating set on tolerance graphs

Dynamic programming:



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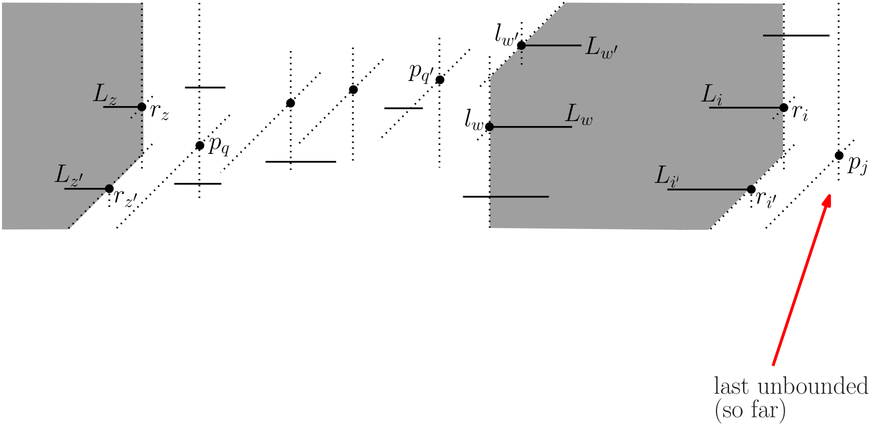

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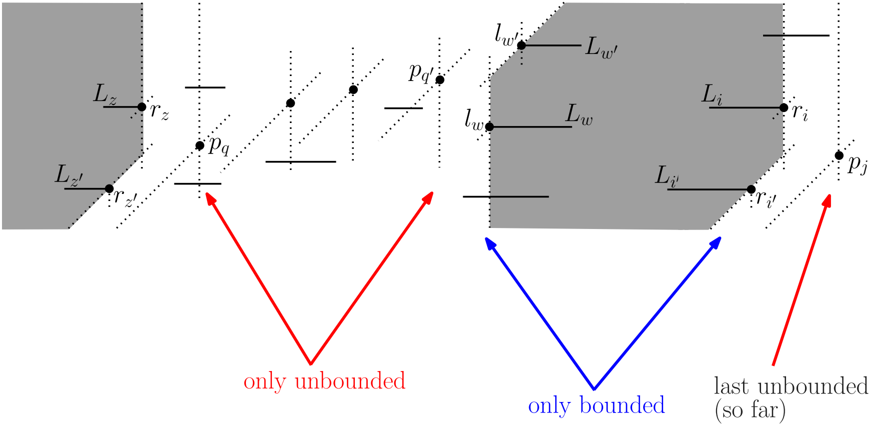

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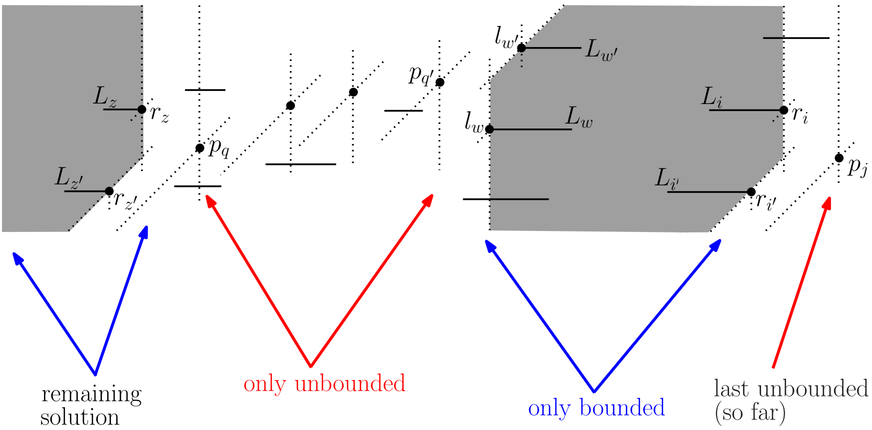

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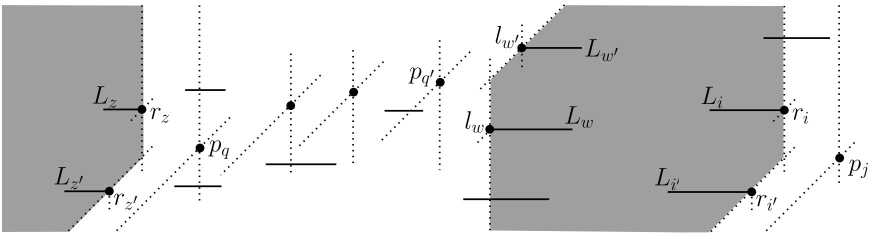



Separate dynamic programming: "bounded" dominating set

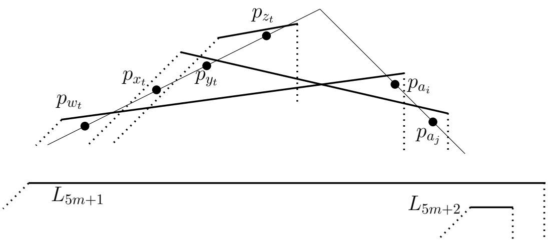
- use only bounded vertices to dominate the (sub)graph
- specifying the "leftmost" and "rightmost" bounded vertices

⟶ not always possible to find a feasible solution !

# Dominating set on multitolerance graphs

On a general (non-horizontal) shadow representation:

- domination set is APX-hard
- reduction from SPECIAL 3-SET COVER (special case of the set cover problem)
- heavily use the different slopes of the line segments
- the spirit of the reduction is inspired from: [Chan, Grant, *Comp. Geometry*, 2014]



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In contrast to dominating set:

- independent dominating set is polynomial on multitolerance graphs
- Sweep line algorithm from right to left

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- Recognition of unit / proper (multi)tolerance graphs ?

# Thank you for your attention!