Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
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NAME: STUDENT NUMBER:

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Algebra III - Abstraktna algebra

1. (a) Denote by $H_0 = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{Mat}_{2\times 2}(\mathbb{R}) \mid a+b+c+d=0\}$ and $H_1 = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{Mat}_{2\times 2}(\mathbb{R}) \mid a+b+c+d=1\}$ given sets. Check is it true that H_0 (respectfully H_1) form a group with respect to operation addition of matrices. Explain your answer very carefully! (50%)

(b) Let H and K denote finite subgroups of G, and let $\gcd(|H|,|K|)$ be prime number. Show that $H \cap K$ is a cyclic group. (50%)

2. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 4 & 8 & 6 & 7 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 7 & 8 & 6 & 5 & 4 & 2 \end{pmatrix}$ be a given permutations.

(a) Write $\alpha\beta$ and $\beta^2\alpha$ as product of disjoint cycles. (25%)

(b) Write $\alpha\beta$ and $\beta^2\alpha$ as product of 2-cycles (that is as product of transpositions). (25%)

(c) Find α^{-1}, β^{-1} and $(\alpha\beta)^{456}$. (50%)

3. (a) Find all homomorphisms from $\mathbb{Z}_4$ to $\mathbb{Z}_2 \times \mathbb{Z}_2$. (50%)

(b) Find group of permutations, which is isomorphic with $\mathbb{Z}_4$. Write down Cayley table for $\mathbb{Z}_4$ and for obtained group of permutations. (50%)

4. Let G denote a given group and let $|G| = 8$. Show that G must have an element of order 2.

Instructions: Exam write only with ink or ballpoint pen in blue or black colour. This sheet of paper you must hand over back together with solutions. Also, all pages that you would like to hand over, mark on the following way: "page-number/number-of-pages".