Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
Koper, 16.02.2017.

NAME: ____________________ STUDENT NUMBER: ____________________

SURNAME: ____________________ SIGNATURE: ____________________

Algebra III - Abstraktna algebra

1. (a) Show that $\mathcal{A} = \{\begin{bmatrix} x & y \\ 0 & 1 \end{bmatrix} \mid x, y \in \mathbb{R},\ x \neq 0\}$ is closed under the operation of (usual) matrix multiplication. Is this operation commutative on the set $\mathcal{A}$? Does every element from $\mathcal{A}$ has an inverse? Carefully explain your answers! (70%)

(b) Let $(\{f : \mathbb{R} \to [0,1]\}, \circ)$ be given semigroup. Find all left and right identity elements. (Recall: A semigroup is an algebraic structure consisting of a set together with an associative binary operation.) (30%)

2. Suppose that A and B are finite cyclic groups with $|A| = n$ and $|B| = m$.

(a) If $n = 2$ and $m = 3$, prove that $A \times B$ is cyclic.

(b) If $n = 2$ and $m = 2$, prove that $A \times B$ is not cyclic.

(c) Is $A \times B$ cyclic if $n = 4$ and $m = 6$?

3. Let $G = \{z \in \mathbb{C} \mid |z| = 1\}$ be a group with respect to multiplication of complex numbers, and let $H = \{1, -1\} \subseteq G$ be a subset of G. Show that H is normal subgroup of G, and show that $G/H \cong G$. [Hint: you can use the first isomorphism theorem.]

4. Prove that a group of order 63 must have an element of order 3.

(a) Using Cauchy's theorem. (10%)

(b) Without using Cauchy's or Sylow's theorems. (90%)

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".