Univerza na Primorskem
Fakulteta za matematiko, naravoslovje in informacijske tehnologije
Koper, 01.12.2017.

NAME: ____________________ STUDENT NUMBER: ____________________

SURNAME: ____________________ SIGNATURE: ____________________

Algebra III - Abstract algebra

1. (a) Let G denote a given group of order n. Show that every element from G appears exactly once in every row and in every column of its Cayley table. (30%)

(b) Let $\mathcal{A} = \{A, B, C, D, P, Q, R\}$ be a given set, and define operation $*$ on a set $\mathcal{A}$, such that $\mathcal{A}$ is a group with respect to operation $*$. Fill in the table on the right side, such that $\mathcal{A}$ form an Abelian group. (70%)

*	A	B	C	D	P	Q	R
A	A	B	C	D	P	Q	R
B	B					R	
C	C				R		
D	D			R			
P	P		R				
Q	Q	R				D	P
R	R					P	Q

2. Let $G = (\mathbb{Z}_{126}, +)$ be a given group.

(a) Find all possible orders of elements. Give an element for each order. (30%)

(b) Count the number of elements for each order. (40%)

(c) Find the number of (cyclic) subgroups of G of order 7 (explain your answer without using known theorems from lectures and exercises). (30%)

3. Let S_5 be given symmetric group (group of all permutations of set $\{1, 2, 3, 4, 5\}$).

(a) Show that S_5 in not an Abelian group. (10%)

(b) Find the number of elements of order 3 in S_5. (30%)

(c) Find the number of elements of order 2 in S_5 (30%)

(d) Find the order of all $5! = 120$ elements from S_5. (30%)

4. Let $f : G \longrightarrow G'$ be a homomorphism of groups and assume that $H \leq G$. Show that

$$f(H) \leq G'.$$

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.