Univerza na Primorskem
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Algebra II - Linear algebra, exam

1. Let $\mathcal{S} = \{e_1 = (1,0,0)^\top, e_2 = (0,1,0)^\top, e_3 = (0,0,1)^\top\}$ denote standard basis of space $\mathbb{R}^3$, and let $v_1 = 2e_1 + (2-b)e_2 + ae_3$, $v_2 = ae_1 + be_2 + 3e_3$, $v_3 = e_1 + e_2 + e_3$ be a given vectors. Discuss and carefully explain for which values of parameters $a, b \in \mathbb{R}$ the set $\{v_1, v_2, v_3\}$ will be a basis of $\mathbb{R}^3$. For $a = 1$ and $b = 2$ write $(1,0,1)^\top$ in the basis $\{v_1, v_2, v_3\}$.

2. Let $\mathcal{V}$ denote some vector space. Recall that $v + \mathcal{U}$ for some $v \in \mathcal{V}$ and some subspace $\mathcal{U}$ of $\mathcal{V}$ is defined as subset of $\mathcal{V}$ by

$$v + \mathcal{U} := \{v + u \mid u \in \mathcal{U}\}.$$

Suppose $\mathcal{U}$ is a subspace of $\mathcal{V}$ and $v, w \in \mathcal{V}$. Show that then the following (i)–(iii) are equivalent.

(i) $v - w \in \mathcal{U}$.

(ii) $v + \mathcal{U} = w + \mathcal{U}$.

(iii) $(v + \mathcal{U}) \cap (w + \mathcal{U}) \neq \emptyset$.

3. Let

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

denote a matrix of linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ in a basis $\{2e_1 - e_2, 2e_2 - e_1\}$. Find the matrix of operator T in a basis $\{e_1 + e_2, 3e_2\}$.

4. Linear operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined on the following way

$$T(x_1, x_2, x_3) = (2x_1 - x_2, x_1 + 2x_2 + x_3, -x_1 + x_2 + x_3).$$

Find all eigenvalues of T. Is it possible to diagonalize T? If answer is positive, find a basis for which T has a diagonal matrix.

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: "page-number/number-of-pages".