Univerza na Primorskem
Fakulteta za matematiko, naravoslovje
in informacijske tehnologije
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NAME: STUDENT NUMBER:

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Algebra II - Linear algebra, 2nd midterm

1. For the following linear transformation T, determine whether T is invertible and justify your answer

$$T : \mathrm{Mat}_{2\times 2}(\mathbb{R}) \to \mathrm{Mat}_{2\times 2}(\mathbb{R}), \qquad T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & a \\ c & c+d \end{pmatrix}.$$

2. Find a polynomial $q \in \mathcal{P}_2(\mathbb{R})$ such that

$$p(\frac{1}{2}) = \int_0^1 p(x)q(x)\,\mathrm{d}x$$

for every $p \in \mathcal{P}_2(\mathbb{R})$.

3. Let $V = C([-1,1])$. Suppose that W_e and W_o denote the subspaces of V consisting of the even and odd functions, respectively.

(a) Prove that V is direct sum of subspaces W_e and W_o.

(b) Prove that $W_e^{\perp} = W_o$, where the inner product on V is defined by

$$\langle f, g\rangle = \int_{-1}^1 f(t)g(t)\,\mathrm{d}t.$$

4. Find value of parameter a such that

$$A = \begin{pmatrix} -4 & 0 & a \\ -14 & 2 & 14 \\ -8 & 0 & 10 \end{pmatrix}$$

as eigenvalues has 2 and 4. For such a diagonalize given matrix i.e. find invertible matrix P, with entries from $\mathbb{R}$, for which $P^{-1}AP = D$ hold, where D is diagonal matrix with entries from $\mathbb{R}$.

Instructions: Please, write your solutions only with ink or ballpoint pen in blue or black colour. You must return this sheet of paper together with your solutions. You can use calculator. All pages with your solutions must be marked in the following way: ”page-number/number-of-pages”.