2 Homework 2 (Four fundamental subspaces)

1. Let $(4,3,2,1)^\top \in \mathbb{R}^4$ be a given vector and let

$$A = \begin{bmatrix} a & -1 & 0 & 0 \\ a & b & -1 & 0 \\ a & 0 & b & -1 \\ a & 0 & 0 & b \end{bmatrix}$$

denote a given matrix. Discus (and carefully explain) for which values of parameters a and b we have $(4,3,2,1)^\top \in \operatorname{im}(A)$.

2. Consider vector subspace of $\mathbb{R}^4$ spanned by vectors $x_1 = (-1,0,1,2)^\top$, $x_2 = (1,2,-3,5)^\top$ and $x_3 = (1,4,0,9)^\top$. Find system of homogeneous linear equations for which space of solution is exactly subspace of $\mathbb{R}^n$ spanned by above three given vectors.

3. Explain is the set, which contains columns of matrix A, linearly independent set, if we have that

$$A = \begin{bmatrix} 7 & 3 & 0 & ... & 0 & 0 \\ 2 & 7 & 3 & ... & 0 & 0 \\ 0 & 2 & 7 & ... & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & ... & 7 & 3 \\ 0 & 0 & 0 & ... & 2 & 7 \end{bmatrix}_{n\times n}$$

(solve the problem without computing $\det(A)$). Is the matrix A a singular matrix? (Recall: square matrix with no inverse is called a singular matrix.)

4. Find for which value of unknown x will vector $(0,1,1,4)^\top \in \mathbb{R}^4$ belong to $\operatorname{im}(A)$ if

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1-x & 1 & 1 & 1 \\ 0 & 1-x & 1 & 1 \\ 0 & 0 & 1-x & 1 \end{bmatrix}.$$