13 Homework 13 (Elementary Properties of Eigensystems & Generalized Eigenvectors and Nilpotnent Operators)

1. Find eigenvalues, eigenspaces and algebraic and geometrical multiplicities of all eigenvalues of A, if matrix A is given by

$$A = \begin{bmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{bmatrix}.$$

2. Let

$$A = \begin{pmatrix} 1 & a & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & b & 0 & 1 \end{pmatrix}$$

be a given matrix. Find parameters a and b if it is known that A is singular matrix which all eigenvalues have algebraic multiplicity 2.

3. Let a denote some real number. Find all eigenvalues and corresponding eigenspaces for a given matrix of n-th order

$$\begin{bmatrix} 1+a & 1 & 1 & \dots & 1 & 1 \\ 1 & 1+a & 1 & \dots & 1 & 1 \\ 1 & 1 & 1+a & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1+a & 1 \\ 1 & 1 & 1 & \dots & 1 & 1+a \end{bmatrix}.$$

4. It is given a matrix $A = \begin{pmatrix} 7 & -4 & 0 \\ a & -7 & b \\ 3 & -2 & 0 \end{pmatrix}$ which eigenvalues are -1 and 1. Find a parameters $a, b \in \mathbb{R}$ and find algebraic and geometrical multiplicities of all eigenvalues of A.

5. Find a real number λ such that $A = \begin{pmatrix} i & 1 \\ 2i & \lambda \end{pmatrix}$ has eigenvector $\begin{bmatrix} i \\ 1 \end{bmatrix}$. Is it possible to diagonalize A?

6. Define $T \in \mathcal{L}(\mathbb{C}^3)$

$$T(z_1, z_2, z_3) = (4z_2, 0, 5z_3).$$

(a) Find all eigenvalues of T, the corresponding eigenspaces, and the corresponding generalized eigenspaces.

(b) Show that $\mathbb{C}^3$ is the direct sum of generalized eigenspaces corresponding to the distinct eigenvalues of T.

7. Define $T \in \mathcal{L}(\mathbb{C}^2)$ by

$$T(w, z) = (z, 0).$$

Find all generalized eigenvectors of T.